Concours interne de recrutement d’administrateurs de l’Insee

Épreuve orale d’admission, sujet « mathématiques et statistiques »

Le candidat dispose de 45 minutes pour préparer le sujet proposé.

Chaque sujet comporte deux exercices à traiter.

L'entretien, d'une durée totale de 45 minutes, se décompose en deux temps: le candidat dispose d'abord de 35 minutes pour présenter les résultats auxquels il est parvenu puis, au cours des 10 dernières minutes, des questions, sur des thèmes a priori différents de ceux déjà abordés, seront proposées au candidat.

Exercice 1

Soit $(a_n)_{n\in\mathbb{N}}$ une suite à termes positifs telle que $\lim\limits_{n\to+\infty} a_n = \ell$.
On pose, pour tout entier naturel n non nul :

$$u_n = \frac{a_n}{a_n + a_{n-1}}$$

1. Montrer que si $\ell \neq 0$ la série de terme général u_n diverge.

2. Montrer que si la série de terme général a_n diverge, alors la série de terme général u_n diverge.
3. Étudier les cas $a_n = \dfrac{1}{2^n}$ et $a_n = \dfrac{1}{2^{n^2}}$.
   Conclusion ?
4. On pose $v_n = \dfrac{a_n}{a_{n-1}}$.
   Montrer que la série de terme général u_n est convergente si et seulement si la série de terme général v_n est convergente.

Exercice 2

On considère une suite de réels $(x_i, i = 1, \ldots, n)$ s'interprétant comme les observations (*déterministes*) d'une variable d'intérêt sur des individus notés i.

Pour des raisons de confidentialité, on ne peut pas publier les données individuelles x_i mais on est amené à les « *flouter* », c'est-à-dire à travailler sur des variables aléatoires de la forme : $Y_i = x_i + \epsilon_i$, où les ϵ_i sont des variables aléatoires centrées, indépendantes et de même loi normale de variance $\sigma^2 > 0$.

1. On considère la somme pondérée (aléatoire) : $Z = \sum_{i=1}^{n} \omega_i Y_i$, où les ω_i sont des *variables aléatoires réelles* (*de signes quelconques*) que l'on astreint à des conditions d'optimalité :
   - **on cherche à assurer la condition :** $\sum_{i=1}^{n} \omega_i Y_i = \sum_{i=1}^{n} x_i$ **(C)**
   - **et à minimiser l'écart des** ω_i **par rapport à 1.**

   a) Montrer que, sous la condition **(C)** :
   $$\sum_{i=1}^{n} (\omega_i - 1)^2 \geq \frac{T_\epsilon^2}{\sum_{i=1}^{n} Y_i^2}, \quad \text{où : } T_\epsilon = \sum_{i=1}^{n} \epsilon_i.$$

   b) En déduire la valeur des ω_i vérifiant la condition **(C)** et minimisant $\sum_{i=1}^{n} (\omega_i - 1)^2$. On notera ces variables ω_i^* et on les exprimera en fonction des x_i et des ϵ_i.

2. Soit i **fixé**.

   a) Étudier la convergence en probabilité de ω_i^* quand *n* tend vers $+\infty$.
   b) Étudier la convergence en loi de $\sqrt{n}\,(\omega_i^* - 1)$ quand *n* tend vers $+\infty$.
   On fera dans les deux cas l'hypothèse que : $\frac{1}{n}\sum_{i=1}^{n} x_i^2 \to l > 0$ *quand n tend vers* $+\infty$.

3. <u>Généralisation du 1</u>.

   Soient deux variables aléatoires X et U de carré intégrable. On pose : $Y = X + U$.

   a) Déterminer une variable aléatoire Ω de carré intégrable telle que :
   $$E(\Omega Y) = EX \quad \text{et minimisant} \quad E(\Omega - 1)^2.$$

   b) Calculer la valeur du minimum obtenu.

   c) Mêmes questions quand on impose les conditions : $E(\Omega Y) = EX$ et $E\Omega = 1$.

Exercice 1

Soit f une fonction continue de $\mathbb{R}^+$ dans $\mathbb{R}^+$, *décroissante et de limite nulle*.
On veut montrer que l'intégrale $\int_0^{+\infty} f(t)\sin t\, dt$ est **convergente**.

1. On pose : $u_n = \int_{n\pi}^{(n+1)\pi} f(t)\sin t\, dt$.
   a) Montrer que : $2f[(n+1)\pi] \le |u_n| \le 2f(n\pi)$.
   b) En déduire que la suite $|u_n|$ est décroissante, de limite nulle.
   c) En déduire la convergence de la série $\sum u_n$.

2. Montrer la convergence de l'intégrale $\int_0^{+\infty} f(t)\sin t\, dt$.

Exercice 2

On considère une suite de variables aléatoires indépendantes, $(X_n)_{n\in\mathbb{N}}$, suivant toute la même loi qu'une variable X, admettant comme densité la fonction f_X définie par :

$$f_X(t) = \begin{cases} 2t \text{ si } 0 \leqslant t \leqslant 1 \\ \\ 0 \text{ sinon} \end{cases}$$

Les trois questions suivantes sont indépendantes

1. **Soit $(a_n)_{n\in\mathbb{N}}$ une suite réelle d'élément de $[0, 1[$ telle que $\lim\limits_{n\to+\infty} a_n = 1$.**
   On définit la variable aléatoire T_n par :

   $$\forall\omega \in \Omega, \quad T_n(\omega) = \min\{i \in \mathbb{N}\ ;\ X_i(\omega) > \sqrt{a_n}\}$$

   Étudier la convergence en loi de la suite $((1 - a_n)T_n)_{n\in\mathbb{N}}$.
2. (a) **Soit ε un réel strictement positif.**
   Déterminer $\lim\limits_{n\to+\infty} \mathbb{P}\left(\sum\limits_{i=0}^{n-1} X_i > \dfrac{2n}{3} + n\varepsilon\right)$.

   (b) **Déterminer $\lim\limits_{n\to+\infty} \mathbb{P}\left(\sum\limits_{i=0}^{n-1} X_i \leqslant \dfrac{2n}{3} + \sqrt{n}\right)$.**
3. **Soit α un réel strictement positif.**
   Étudier, suivant les valeurs de α, la convergence en loi de la suite $(R_n)_{n\in\mathbb{N}^*}$ définie par :

   $$R_n = \min(n^\alpha X_0, n^\alpha X_1, \dots, n^\alpha X_{n-1})$$

Concours interne de recrutement d’administrateurs de l’Insee

Épreuve orale d’admission, sujet « économie »

Le candidat dispose de 45 minutes pour préparer l’un de ces trois sujets.

Chaque sujet comporte deux exercices à traiter.

Sujet 1

I – Production domestique

Un ménage comprend un seul individu. Sa fonction de production domestique est donnée par :

$$y = f(h,x) = hx$$

où y est la quantité d'un bien domestique agrégé produite à partir de la fonction de production f, h représente le temps de travail domestique et x la quantité agrégée de biens marchands, de prix unitaire égal à p_x, utilisés dans la production domestique. Soit w le taux de salaire de l'individu(e) s'il/elle travaille sur le marché.

1) A quelle catégorie la fonction de production appartient-elle ? Tracez quelques isoquantes. Sont-elles convexes ? Quelle(s) autre(s) propriétés possèdent-elles ? (3 pts)

2) Déterminez les demandes conditionnelles de facteurs, ainsi que la fonction de coût du ménage. Le lemme de Shephard permet-il ici de retrouver les demandes conditionnelles des deux facteurs ? (3pts)

3) Quelle est la nature des rendements d'échelle de la production ? Supposons que le ménage puisse écouler le bien domestique qu'il produit au prix p sur un marché concurrentiel et cherche à maximiser le profit qu'il en tire. En ignorant les conditions du second ordre de la maximisation du profit, que trouverait-on pour ces différentes fonctions si l'on cherchait à calculer les demandes notionnelles de facteurs, la fonction d'offre de bien domestique par le ménage, et la fonction de profit ? La valeur trouvée pour la fonction de profit doit vous inciter à comprendre qu'il y a un problème dont on expliquera la nature en le reliant à la première partie de la question. (3 pts)

4) Le ménage a-t-il nécessairement intérêt à allouer tout son temps disponible hors loisirs à la production domestique ? A quelles contraintes et à quel type d'arbitrage sera-t-il soumis ? (2 pts)

II – Probabilités

Une pièce a une probabilité p = 1/2 de tomber sur « pile ». On vous propose un jeu dans lequel vous gagnez 2^j euros si « pile » apparaît pour la première fois au jème lancer de la pièce.

1) Calculez l'espérance de gain de ce jeu. (3 pts)

2) Vous avez une fonction d'utilité de type VNM. Précisez de quoi il s'agit. (2 pts)

3) Votre utilité du revenu est u(R) = ln(R). Quelle est votre attitude générale envers le risque ? Exprimez comme une somme l'espérance d'utilité de ce jeu. (2 pts)

4) Calculez l'espérance d'utilité du jeu (on admettra que $\sum_{j=1}^{\infty} \frac{j}{2^{j-1}} = 4$). En déduire la somme pour laquelle vous seriez indifférent(e) entre recevoir cette somme de façon certaine et avoir la possibilité de jouer à ce jeu. Cela vous paraît-il raisonnable ? (2 pts)

Sujet 2

I Equilibre général

On considère une économie d'échange comportant deux consommateurs. Le premier a pour fonction d'utilité $u_1(q_1,z_1) = q_1z_1$. Le second a la même fonction d'utilité : $u_2(q_2,z_2) = q_2z_2$. Les biens q et z sont disponibles dans l'économie en quantité totale Q = 3 et Z = 1.

1) Tracez le rectangle de côté Q = 3 et Z = 1 (appelé boîte d'Edgeworth) à partir duquel on représentera le système de préférences du consommateur 1 avec l'origine en bas à gauche du rectangle et des quantités de bien q et z croissant dans le sens habituel et celui du consommateur 2 en haut à droite du même rectangle avec des quantités de bien q et z croissant dans le sens opposé. Expliquez comment chaque point à l'intérieur de la boîte d'Edgeworth correspond de façon bi-univoque à une répartition possible des entre les deux consommateurs des quantités totales disponibles des deux biens. (3 pts)
2) Représentez dans la boîte d'Edgeworth quelques courbes d'indifférence de chacun des deux consommateurs. (1 pts)
3) On appelle courbe des contrats le lieu des points dans la boîte d'Edgeworth vérifiant l'égalité $TMS_1 = TMS_2$. Exprimer en fonction de q_1 et de z_1 l'équation définissant la courbe des contrats de cette économie, puis l'exprimer sous la forme d'une fonction $z_1 = f(q_1)$ que l'on déterminera et que l'on tracera dans la boîte. (2 pts)
4) Les consommateurs 1 et 2 disposent initialement des allocations suivantes : (Q_1=2; Z_1=1/3) et (Q_2=1; Z_2=2/3). En notant p_q et p_z les prix unitaires respectifs des biens, après avoir exprimé le revenu de chaque consommateur en fonction des prix, écrire les conditions d'équilibre de chacun des deux consommateurs, ainsi que celles de l'économie dans son ensemble, c'est-à-dire celles qui les rendent compatibles compte tenu des quantités disponibles. (2 pts)
5) Déterminez l'équilibre général de cette économie, c'est-à-dire les allocations d'équilibre déterminant à la fois la maximisation de l'utilité de chacun des consommateurs et le rapport des prix. Représentez-le graphiquement. (3 pts)

II – Duopoles

Deux entreprises (A et B) en concurrence produisent un bien homogène. Les deux entreprises ont un coût marginal contant Cm = k. Dans chacune des trois situations suivantes (1) 2) 3)), décrivez comment évoluent la production et les prix si les entreprises sont : (i) à l'équilibre de Cournot (ii) à l'équilibre de collusion et (iii) à l'équilibre de Bertrand.

1) L'entreprise A doit augmenter ses salaires à la suite de quoi son coût marginal passe à 2k (3 pts)
2) Le coût marginal des deux entreprises augmente de la même façon (3 pts)
3) La courbe de demande se déplace vers la droite (3 pts)

Concours interne de recrutement d’administrateurs de l’Insee

Épreuve orale d’admission : anglais

Le candidat dispose de 45 minutes pour commenter un texte abordant un sujet d’ordre général, remis au préalable, et portant sur les domaines économique ou social, suivi d’une interrogation sur les idées principales du texte et d’un échange sur la carrière ou le projet professionnel du candidat.

Exemples de textes pouvant être proposés :

It's Scary How Dominant Apple Has Become. But How Scary?

Farhad Manjoo

How big of a deal is it that a film from Apple, founded as a computer company, just won three Academy Awards, including one for best picture? If you're among those worried about the inescapable economic, political and cultural dominance of that set of nerdy giants we now call Big Tech, how much greater panic should this induce?

Perhaps none. On its own, the awards-season success of "CODA" — the feel-good film about a deaf family that Apple purchased for its subscription TV service at last year's Sundance Film Festival — will be immaterial to the company's very big bottom line. And anyway, Apple is hardly the first corporate interloper to find fortune (or, more often, failure) in showbiz. For decades, auto-parts companies, Canadian liquor magnates and storied American corporations have gone to Hollywood in search of lucrative side hustles and brand-enhancing corporate jewels. Sony, whose Walkman was the iPod of its day, got into the entertainment business way back in 1989; now Sony Pictures' capacious library of films includes a dozen best picture Oscar winners.

Still, when "CODA" won big on Sunday night, I was struck by a set of complicated insights about this biggest of Big Tech behemoths. And I was again left wondering: As a technological and cultural force, should we be more inclined to celebrate Apple or to fear it?

Among my epiphanies were the following.

First: Boy, Apple sure is executing well these days. As recently as the late 2010s, with the Mac line foundering, its cloud services second-rate and each new iPhone hardly better than the last, Apple had seemed to grow a bit lazy, even boring. In the 2020s it's shaken off any such malaise. Over the past year, I have been wowed by a parade of Apple products — the speed of its new Macs, the cameras and battery life on the latest iPhones, the way new versions of iPadOS and MacOS have become magically interoperable, the way Face ID now works with masks. Nowadays, even Siri is sometimes halfway helpful. The wins for "CODA" and Apple's streaming success fit that larger story: This is a very well-run company making very good products that customers are willing to spend a lot of money for.

Second: But a film from Apple becoming the first from a streaming service to win the Oscar for best picture was not just a tale of a well-oiled corporate machine. Apple got into the movie business the way a lot of the tech giants do a lot of things nowadays: It spent gobs of money to win a top spot in a market dominated by much smaller companies, and where money wasn't enough, it used its advantages as a tech platform to help it along. We first saw this sort of bigfooting with music. In 2015, in an effort to compete with Spotify and other music streamers, Apple introduced a music service that came installed on the iPhone and handed out free three-month subscriptions to anyone who wanted one. Early mixed reviews for Apple Music didn't matter; because it was baked into the device, Apple's music plan quickly garnered millions of paying users, and today it reportedly has more subscribers than every rival other than Spotify.

In 2019 it did a similar thing with TV. Apple spent a reported $6 billion on content to start Apple TV+, and it gave away a free year of the service with the sale of new Apple devices. Apple TV+'s lineup was full of sleepers — even "Ted Lasso," its most beloved show, takes some time to warm up to — but with a built-in audience of every new iPhone, iPad and Mac user, the company could afford to take its time to find its footing before asking people to pay. The success of "CODA" was also a story of Apple's deep pockets: The $25 million Apple spent on the film's distribution rights was a Sundance record. According to The Wall Street Journal, a veteran awards consultant estimated that Apple spent more than $10 million on the Oscar campaign for "CODA" — more than the film cost to produce.

Apple can continue to throw money at its TV service indefinitely; it could easily afford to never make any money from TV+ and simply

https://www.nytimes.com/2022/03/31/opinion/apple-coda-oscar.html

run the service as a kind of brand-marketing project. The company's revenue was about $366 billion in fiscal year 2021. Netflix's revenue last year was just under $30 billion — about 8 percent of Apple's.

Third: All of this would seem bad — bad in an antitrust, massive-corporations-gobbling-up-everything sort of way. Netflix and Spotify remain thriving companies, but it just does not seem fair or conducive to competition for Apple to leverage its dominance in one market, smartphones, to get ahead in other markets, like the music and movie businesses. It's especially troublesome when you consider all the onerous rules that Apple imposes on its rivals through its App Store. For instance, it generally takes up to a 30 percent cut of revenue that app makers collect through in-app purchases. Apple's own apps don't have to worry about such concerns.

But again, there are complications here. For one thing, Apple is not, in traditional terms, anything close to a monopoly in the smartphone business. Although analysts believe it makes the vast bulk of the profits in the smartphone industry, its global market share is on par with many rivals'. In 2020, Epic Games, the maker of "Fortnite," sued Apple to fight the 30 percent commission and other App Store rules. Last year Apple largely won the case. "Given the trial record, the court cannot ultimately conclude that Apple is a monopolist under either federal or state antitrust laws," a federal judge ruled. (Both Apple and Epic are appealing the decision.)

The other wrinkle is that Apple sometimes uses its market power in ways that are inarguably good for its customers. The most recent example is App Transparency Tracking, a phenomenal privacy feature that Apple added to iPhones and iPads last year. The system cracked down on rampant privacy abuses by the internet advertising industry. Now when you run Facebook, Instagram, Twitter and many other ad-supported iOS apps, Apple requires the apps to get consent from users to collect some of their information for advertising purposes. Surprise: When you force apps to ask users if they want to be tracked, a lot of people decline. Just how many people became apparent in February, when Facebook's parent company, Meta, said the feature would cost the company $10 billion in revenue this year.

I regret to say that after considering the scope of Apple's power, I've arrived at the sort of no-easy-answer muddle that better opinion columnists than me usually try to avoid. But that's just where we are: Apple, with a valuation of about $3 trillion and firing on every cylinder, seems unstoppable. On the one hand, its market power is scary, and sometimes its ethical and moral compass leaves a lot to be desired. (See its deference to the Chinese government.)

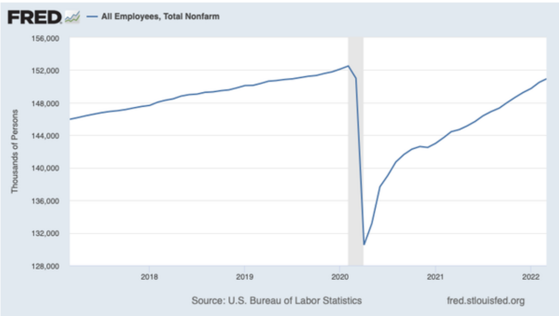
On the other hand, it is probably the best-run, most innovative and still most consumer-friendly of the Big Tech baddies. Maybe that's as good as it gets.

Texte 2

What Ever Happened to the Great Resignation?

Paul Krugman

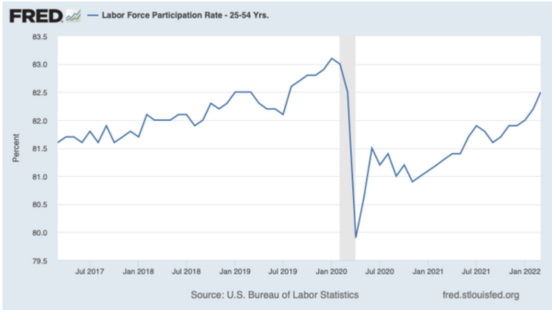
All of the evidence suggests that right now, it's unusually easy for U.S. workers to find jobs and unusually hard for employers to find workers. The odd thing is that we have a very tight labor market, even though the number of employees is still about a million and a half below prepandemic levels and even further below the prepandemic trend:



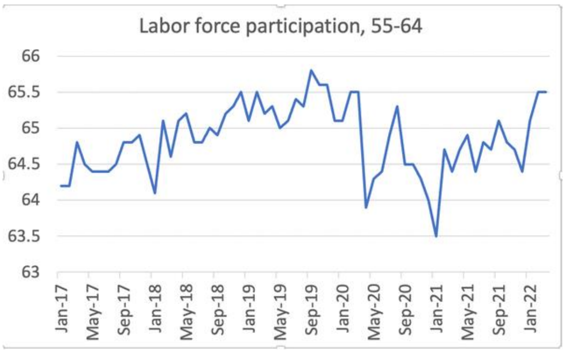
For some time, many people, [myself](#) included, have been telling a story about this situation that goes by the name of the Great Resignation. That tale goes like this: The Covid pandemic caused many Americans to reconsider whether they really wanted or needed to keep working. Fear of infection or lack of child care kept some workers home, where they discovered that the financial rewards of their jobs weren't enough to compensate for the costs of commuting and the unpleasantness of their work environment. Older workers, forced into unemployment, decided that they might as well take early retirement. And so on.

Well, when my information changes, I change my mind — a line often but dubiously attributed to John Maynard Keynes, but whatever. And the past few months of data have pretty much destroyed the Great Resignation narrative.

Have large numbers of Americans dropped out of the labor force — that is, they are neither working nor actively seeking work? To answer this question, you need to look at age-adjusted data; falling labor force participation because a growing number of Americans are over 65 isn't meaningful in this context. So economists often look at the labor force participation of Americans in their prime working years: 25 to 54. And guess what? This participation rate has surged recently. It's still slightly below its level on the eve of the pandemic, but it's back to 2019 levels, which hardly looks like a Great Resignation:



What about early retirement? If a lot of that was happening, we'd expect to see reduced labor force participation among older workers, 55 to 64. But they've come rapidly back into the labor force:



A few months ago, it still seemed reasonable to talk about a Great Resignation. At this point, however, there's basically nothing there. It's true that an unusually high number of workers have been quitting their jobs, but they have been leaving for other, presumably better jobs, rather than leaving the work force. As the labor economist Arindrajit Dube says, it's more a Great Reshuffling than a Great Resignation.

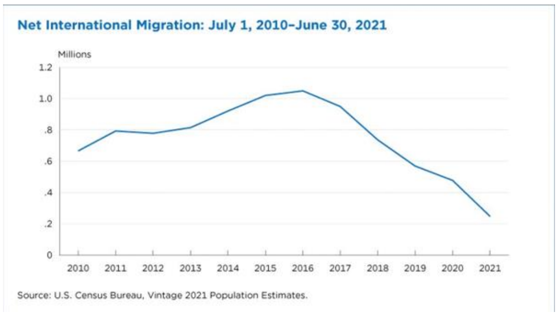
Yet if workers have for the most part come back to the labor force, how do we explain the seeming paradox with which I began this newsletter? How can labor markets be so tight when payroll employment is still well below the prepandemic trend?

https://www.nytimes.com/2022/04/05/opinion/great-resignation-employment.html

I'm sure that labor economists are scrambling to figure this out properly, but a quick look at the evidence suggests a couple of factors that many people telling the Great Resignation narrative — again, myself included — missed.

First, as the economist [Dean Baker] has been pointing out, the most commonly cited measures of employment don't count the self-employed, and self-employment is up by a lot, around 600,000 more workers than the average in 2019. Some of this self-employment may be fictitious — gig workers who are employees in all but name but work for companies that classify them as independent contractors to avoid regulation. But it also does seem as if part of the Great Reshuffling has involved Americans concluding that they could improve their lives by starting their own businesses.

Second, a point that receives far less attention than it should is the decline of immigration since Donald Trump came to office, which turned into a plunge with the coming of the pandemic:



Many immigrants are working age and highly motivated; their absence means that we shouldn't have expected employment to maintain its old trend.

Does the declining plausibility of the Great Resignation narrative have any policy implications?

Well, I don't like saying this, but it does seem to reinforce the case for higher interest rates. Until recently, it was fairly common for monetary doves to argue that we weren't really at full employment, because there were many potential workers still sitting on the sidelines. That's now a hard case to make; the U.S. economy now looks overheated by just about every measure, which means that it needs to be cooled off a bit.

The other implication is that if we want to revive U.S. economic vitality, we really should try to re-establish our nation's historic role as a destination for ambitious immigrants. But that's not a policy idea likely to get much traction, given the American right's anti-immigrant hysteria.

Anyway, you should know that all of those stories about how Americans are no longer willing to work seem to have evaporated. The Great Resignation now looks like a Great Misunderstanding.

https://www.nytimes.com/2022/04/05/opinion/great-resignation-employment.html