Online complement C1 – The Mélèze model

Households

Consumption and investment decision of Ricardian households

In both regions, we assume that a fraction $(1 - \mu^i)$ of households can participate to the financial markets. These households can borrow or lend money on an international market and doing so have the possibility to smooth their consumption across periods. Households of this type (τ) maximize their intertemporal utility function subject to their budget constraint. Utility is non-separable, CES in consumption with external habit formation in a multiplicative manner. This functional form stems from Trabandt & Uhlig (2011) and is compatible with long term growth (King et al., 2002), as under this form the disutility of labor is concave for any value of the intertemporal elasticity of substitution of consumption, and also ensures a constant Frisch elasticity. The representative Ricardian household solves:

$$\max_{C_T^{R,i}, FA_T^i, I_T^i, K_T^i} E_t \sum_{T=t}^{\infty} \beta^{i^{T-t}} \mathcal{U}\left(\frac{C_t^{R,i}}{(1-\mu^i)n^i\mathbb{N}}, C_{T-1}^i\right) \mathcal{V}\left(\frac{l_T^{R,i}}{(1-\mu^i)n^i\mathbb{N}}, L_{T-1}^i\right)$$

subject to the budget constraint

$$FA_{T}^{i} = \left(R_{T-1} - \psi(\frac{FA_{T-1}^{i}}{P_{T-1}^{i}\bar{Y}^{i}})\right)FA_{T-1}^{i} + w_{T}^{i}l_{T}^{R,i} - CPI_{T}^{i}(1 + \nu_{T}^{c,i})C_{T}^{R,i} + (1 - \nu_{T}^{D,i})D_{T}^{i} + (1 - \nu_{T}^{K,i})CPI_{T}^{i}r_{T}^{K,i}K_{T-1}^{i} - CPI_{T}^{i}(1 + \nu_{T}^{c,i})I_{T}^{i}$$

and the capital accumulation equation

$$K_T^i = (1 - \delta) K_{T-1}^i + \left[1 - \delta \left(\frac{\varepsilon_T^{i,I} I_T^i}{I_{T-1}^i} \right) \right] I_T^i$$

Under the most general form, we define utility as:

$$\begin{aligned} \mathcal{U}\left(\frac{C_{t}^{R,l}}{(1-\mu^{i})n^{i}\mathbb{N}}, C_{t-1}^{i}\right)\mathcal{V}(L_{t}^{R,i}, L_{t-1}^{i}) &= \\ \frac{1}{1-\sigma_{c}^{i}}\left(\frac{C_{t}^{R,i}}{(1-\mu^{i})n^{i}\mathbb{N}}\left(\frac{C_{t-1}^{i}}{n^{i}\mathbb{N}}\right)^{-h_{c}^{i}}\right)^{1-\sigma_{c}^{i}}\left(1-\kappa^{i}\varepsilon_{t}^{L,R,i}(1-\sigma_{c}^{i})\left(\frac{L_{t}^{R,i}}{(1-\mu^{i})n^{i}\mathbb{N}}\right)^{1+\sigma_{l}^{i}}\right)^{\sigma_{c}^{i}} \end{aligned}$$

N is the total population and n^i the share located in region *i*. E_t , β^i are respectively the expectation at time *t* operator and the discount factor, $C_t^{R,i}$ is the aggregate consumption of Ricardian households in region *i*, σ_c^i is the inverse intertemporal elasticity of substitution. κ^i is the weight assigned to labor in the utility function, σ_l^i is the inverse of the Frisch elasticity. h_c^i are the external habit formation parameters on consumption. $l_t^{R,i}(\tau)$ is the labor supply of household τ , $w_t^i(\tau)$ its wage and $\varepsilon_t^{L,R,i}$ a labor supply shock.

 FA_t^i is the aggregate level of private financial assets, R_t is the interest rate set by the monetary authority in the union, ψ is an interest premium on debt (where \overline{Y}^i corresponds to the steady state level of output). This cost is introduced to ensure the stationarity of the model (i.e. rule out unit roots). This premium is akin to a transaction cost on holding assets and is paid to an international financial intermediary. $v_t^{c,i}$ is the tax rate on consumption or value-added tax (VAT) through which government expenditure is partially financed. D_t^i is the dividend paid by the firm to its owners taxed at rate $v_t^{D,i}$, FD_t^i are equivalently the dividends paid by the financial sector taxed at rate $v_t^{FD,i}$ and Φ_t^i a lump-sum transfer from the government. Finally, K_t^i is the capital stock of *Ricardian* households depreciating at rate δ and which revenues are taxed at rate $v_t^{K,i}$.

In the capital accumulation equation, $I_t^i(\tau)$ is the investment level with an adjustment cost $\mathcal{S}(\varepsilon_T^{i,I}I_T^iI_{T-1})$ depending on previous period level of investment. As in Smets & Wouters (2003) to Smets & Wouters (2007), we assume that at steady state S=0, S'=0 and S">0. $\varepsilon_T^{i,I}$ represents an exogenous cost-push shock. The costate variable for the capital accumulation constraint is defined as $q_t CPI_t^i(1 + v_t^{c,i})$ times the costate variable of the budget constraint $\beta^{i^t}\lambda_t$, so that q_t is the market value of an additional unit of capital, that is Tobin's marginal Q.

We define the stochastic discount factor between t and t + 1 as follows:

$$\mathcal{Q}_{t,t+1}^{R,i} = \frac{\mathcal{U}'\left(\frac{C_{t+1}^{R,i}}{(1-\mu^{i})n^{i}\mathbb{N}}, C_{t}^{i}\right)\mathcal{V}\left(\frac{L_{t+1}^{R,i}}{(1-\mu^{i})n^{i}\mathbb{N}}, L_{t}^{i}\right)}{\mathcal{U}'\left(\frac{C_{t}^{R,i}}{(1-\mu^{i})n^{i}\mathbb{N}}, C_{t-1}^{i}\right)\mathcal{V}\left(\frac{L_{t}^{R,i}}{(1-\mu^{i})n^{i}\mathbb{N}}, L_{t-1}^{i}\right)}$$

As a result, the Euler equation writes:

$$\beta^{i}E_{t}\left\{\mathcal{Q}_{t,t+1}^{R,i}\frac{R_{t}-\psi\left(\frac{FA_{t}^{i}}{P_{t}^{i}\bar{Y}^{i}}\right)}{\prod_{t+1}^{c,i}\frac{1+\nu_{t+1}^{c,i}}{1+\nu_{t}^{c,i}}}\right\}=1$$

where $\Pi_{t+1}^{c,i}$ is the inflation of the consumption price index in region *i*. Investment and the marginal value of capital are described by the following first order conditions:

$$1 = q_{t}^{i} \left\{ 1 - S\left(\frac{\varepsilon_{t}^{i,I}I_{t}^{i}}{I_{t-1}^{i}}\right) - S'\left(\frac{\varepsilon_{t}^{i,I}I_{t}^{i}}{I_{t-1}^{i}}\right) \frac{I_{t}^{i}}{I_{t-1}^{i}} \right\} + \beta^{i}E_{t} \left\{ \mathcal{Q}_{t,t+1}^{R,i}q_{t+1}^{i}\varepsilon_{t+1}^{i,I}S'\left(\varepsilon_{t+1}^{i,I}\frac{I_{t+1}^{i}}{I_{t}^{i}}\right)\left(\frac{I_{t+1}^{i}}{I_{t}^{i}}\right)^{2} \right\}$$
$$q_{t}^{i} = \beta^{i}E_{t} \left\{ \mathcal{Q}_{t,t+1}^{R,i}\left(q_{t+1}^{i}(1-\delta) + \frac{(1-\nu_{t+1}^{k,i})r_{t+1}^{k,i}}{1+\nu_{t+1}^{c,i}}\right) \right\}$$

The latter, similar to the Euler equation on consumption, describes the trade-off between investment in capital and consumption.

Consumption decision of non-Ricardian households

The remaining fraction μ^i of households does not have access to financial intermediaries and therefore, their consumption cannot be smoothed across periods. These *non-Ricardian* households follow a rule-of-thumb:

$$0 = w_t^i(\tau) l_t^i(\tau) + \Phi_t^i(\tau) - CPI_t^i(1 + v_t^{c,i})C_t^i(\tau)$$

on aggregate
$$0 = W_t^{NR,i} L_t^{NR,i} + \Phi_t^{NR,i} - CPI_t^i(1 + v_t^{c,i})C_t^{NR,i}$$

Labor supply decision and wage setting

Labor is assumed immobile across regions. Besides, we assume wage stickiness à la Calvo (1983), with parameter ξ_w^i denoting the probability not to adjust wages at each period. There is also partial indexation of wages on past inflation of consumption prices according to parameter γ_w^i and indexation on targeted inflation with parameter $1 - \gamma_w^i$. In addition, wages are also indexed on the deterministic trend of TFP. These indexations are necessary to ensure that the distribution of wages does not diverge when there is non zero inflation and exogenous growth at steady state. A given household τ solves the following program:

$$\max_{\widetilde{w}^{i}_{t}(\tau), \widetilde{l}^{i}_{t,T}(\tau)} E_{t} \sum_{T=t}^{\infty} \left(\xi^{i}_{w} \beta^{i} \right)^{T-t} \mathcal{U}(C^{i}_{T}(\tau), C^{i}_{T-1}) \mathcal{V}\left(\widetilde{l}^{i}_{t,T}(\tau), L^{i}_{T-1} \right)$$

subject to the labor demand function:

$$\tilde{l}_{t,T}^{i}(\tau) = \frac{1}{n^{i}\mathbb{N}} \left(\frac{\widetilde{w}_{t,T}^{i}(\tau)}{W_{T}^{i}}\right)^{-\theta_{W}^{i}} L_{T}^{i}$$

as well as their respective budget constraint, and the following indexation rule:

$$\widetilde{w}_{t,T}^{i}(\tau) = \widetilde{w}_{t}^{i}(\tau) \prod_{k=t}^{T-1} (\Pi_{k}^{c,i})^{\gamma_{w}^{i}} (\overline{\Pi}^{c,i})^{1-\gamma_{w}^{i}} = \widetilde{w}_{t}^{i}(\tau) \Gamma_{w,t}^{T-1}$$

where $\widetilde{w}_{t}^{i}(\tau)$ is the optimal wage set at time t by household τ and $\widetilde{w}_{t,T}^{i}(\tau)$ is its wage at time T when not reset between time t and T, $\tilde{l}_{t}^{i}(\tau)$ and $\tilde{l}_{t,T}^{i}(\tau)$ are the corresponding labor demands. $\Gamma_{w,t}^{T-1}$ denotes the indexation factor $\prod_{k=t}^{T-1} (\prod_{k=1}^{c,i})^{\gamma_{w}^{i}} (\overline{\Pi}^{c,i})^{1-\gamma_{w}^{i}}$ with $\overline{\Pi}^{c,i}$ the steady state inflation of CPI^{i} .

The aggregate first order condition reads

$$0 = E_{t} \sum_{T=t}^{\infty} \left(\xi_{w}^{i} \beta^{i} \right)^{T-t} \tilde{l}_{t,T}(\tau) \mathcal{Q}_{t,t+1}^{R,i} \left[\frac{\mathcal{U}(C_{T}^{i}(\tau), C_{T-1}^{i}) \mathcal{V}'\left(\tilde{l}^{i}_{t,T}(\tau), L_{T-1}^{i} \right)}{\mathcal{U}'(C_{T}^{i}(\tau), C_{T-1}^{i}) \mathcal{V}\left(\tilde{l}^{i}_{t,T}(\tau), L_{T-1}^{i} \right)} + \frac{\theta_{w}^{i} - 1}{\theta_{w}^{i}} \frac{\tilde{w}_{t}^{i}(\tau) \Gamma_{w,t}^{T-1}}{CP I_{T}^{i}(1 + v_{T}^{c,i})} \right]$$

where one may recognize the stochastic discount factor between time t and T and between brackets, the wedge between the ratio of the marginal utility of labor and consumption and the real wage with a term in θ_w^i representing the market power of households. Note that this wage setting equation is at the individual level and therefore that the associated utility function and wages depend on the individual consumption of household τ . Therefore, there are two wage Phillips curves, one for each type of households. In addition, as for the Euler and investment equations, we make the standard assumption that individual dispersion can be neglected (Campagne & Poissonnier, 2016).

Firms

We assume an exogenous and global technological growth process in the form $\zeta_t^i = \varepsilon_t^{\zeta,i} (1+g)^t \bar{\zeta}^i$, where g is the deterministic growth rate of total factor productivity, $\bar{\zeta}$ the *de-trended* steady state level of technology, and ε_t^{ζ} a stochastic productivity shock. We assume that technology can be shared and transferred within the union, so that TFP growth is the same in both regions. However, the steady state detrended level of TFP, ie. $\bar{\zeta}^i$, differs across regions to take into account the initial differences in wealth across regions.

Production factors optimization

Firms hire domestic labor at the cost $W_t^i(1 + v_t^{w,i})$, where $v_t^{w,i}$ is the payroll tax rate levied by the government on firms.¹ They also rent capital from households at rate $r^{k,i}$. In real term the rental cost of demanded capital $K_t^{d,i}$ is then $r_t^{k,i}K_t^{d,i}$ paid at time t. In nominal terms, this cost equals $r_t^{k,i}K_t^{d,i}CPI_t^i$: the value of the rented capital in current is equal to the real capital stock times its market price $CPI_t^{i,2}$. Note that capital from previous period is used for production at time assuming installation delays. Therefore, at market equilibrium, we have on aggregate $K_t^{d,i} = K_{t-1}^i$.

In each region *i*, at first order, neglecting price dispersion, firms produce differentiated goods Y_t^i with the following technology:

$$Y_t^i = \left(\zeta_t^i L_t^i\right)^{1-\alpha} \left(K_t^{d,i}\right)^{\alpha} \text{ at cost } W_t^i (1+\nu_t^{w,i}) L_t^i + r_t^{k,i} CPI_t^i K_t^{d,i}$$

where α is the share of capital costs in value added. For the sake of simplicity, firms hire both types of households indistinctly. First order conditions yield:

$$\frac{1-\alpha}{\alpha} = \frac{W_t^i (1+v_t^{w,i}) L_t^i}{r_t^{k,i} K_t^{d,i} CPI_t^i} \text{ on aggregate } \frac{1-\alpha}{\alpha} = \frac{W_t^i (1+v_t^{w,i}) L_t^i}{r_t^{k,i} K_{t-1}^i CPI_t^i}$$

The real marginal cost of production is identical across firms and does not depend on its size:

$$RMC_{t}^{i} = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \left(\frac{RW_{t}^{i}}{\zeta_{t}^{i}} (1+\nu_{t}^{c,i})(1+\nu_{t}^{w,i}) \right)^{1-\alpha} \left(r_{t}^{k,i} \right)^{\alpha} \frac{CPI_{t}^{i}}{P_{t}^{i}}$$

Price setting

The price setting follows Calvo process in each region. Firm ε can reset its price with exogenous probability $(1 - \xi_i)$. Producers know the relationship between their price and the demand for their product and choose their price $\tilde{P}_t^i(\varepsilon)$ so as to maximize their expected profit under that constraint:

$$\max_{\tilde{P}_{t}^{i}(\varepsilon)} E_{T=t} \sum_{T=t}^{\infty} (\beta^{i}\xi^{i})^{T-t} \lambda_{T}^{i} \big(\tilde{P}_{t}^{i}(\varepsilon) \tilde{y}_{t,T}^{i}(\varepsilon) - W_{T}^{i}(1+\nu_{T}^{w,i}) L_{t,T}^{i}(\varepsilon) - r_{T}^{k,i} CP I_{T}^{i} K_{t,T}^{d,i}(\varepsilon) \big)$$

subject to $\tilde{y}_{t,T}^{i}(\varepsilon) = \frac{1}{p^{i}\mathbb{P}} \left(\frac{\tilde{p}_{t,T}^{i}(\varepsilon)}{P_{T}^{i}}\right)^{-\theta^{i}} Y_{T}^{i}$ and previous technological constraints of the firm

$$\tilde{P}_{t,T}^{i}(\varepsilon) = \tilde{P}_{t}^{i}(\varepsilon) \prod_{k=t}^{T-1} (\Pi_{k}^{i})^{\gamma_{p}^{i}} (\overline{\Pi}^{i})^{1-\gamma_{p}^{i}} = \tilde{P}_{t}^{i}(\varepsilon) \Gamma_{t}^{T-1}$$

where the Lagrange multiplier λ_T^i is the marginal utility of a representative *Ricardian* households in region *i*. These households own the firms, so logically their utility enters the price-setting program. This is however neutral on the linearized Phillips curve apart from a redefinition of β when there is long term growth, a redefinition which does not depend on households' type. $\tilde{y}_{t,T}^i(\varepsilon)$ is the demand for goods produced by firm ε of region *i* at time *T* when its price was last reset at time *t*. γ_p^i is the parameter of price indexation on past inflation and Γ_t^{T-1} denotes $\prod_{k=t}^{T-1} \prod_k^i \gamma_p^{i}(\overline{\Pi}^i)^{1-\gamma_p^i}$. So $\tilde{P}_{t,T}^i(\varepsilon) = \tilde{P}_t^i(\varepsilon)\Gamma_t^{T-1}$ is the

¹ No taxes on labor income (social contribution, income tax) are paid by households here. The steady state is not affected by this assumption but the reaction of wages to this tax is affected in the short-term.

 $^{^{2}}$ The price of capital is by convention the same as the price of investment, which is identical to the price of consumption as we assume that both goods are identical.

price of good ε of region *i* at time *T* when its price was last reset at time *t*. Note that Π_t^i is the inflation of goods produced in region *i* and differs from inflation of the consumption price index CPI_t^i , which includes inflation from imported goods as well. $\overline{\Pi}^i$ is the steady state value of Π_t^i .

The first order condition for firm ε reads:

$$0 = \sum_{T=t}^{\infty} \left(\beta^{i}\xi^{i}\right)^{T-t} \lambda_{T}^{i} \frac{Y_{T}^{i}}{p^{i}\mathbb{P}} \left(\frac{\tilde{P}_{t}^{i}(\varepsilon)\Gamma_{t}^{T-1}}{P_{T}^{i}}\right)^{-\theta^{i}} \left(\tilde{P}_{t}^{i}(\varepsilon)\Gamma_{t}^{T-1} - \frac{\theta^{i}}{\theta^{i}-1}MC_{T}^{i}\right)$$

Dividends distribution

Firms cannot save or invest, so they distribute their profits to households. This distribution can be thought of as dividends to firms' owners. We assume that only unconstrained households, who have access to financial and investment markets, are paid such dividends D_t^i .

$$D_t^i = P_t^i Y_t^i - W_t^i (1 + v_t^{w,i}) L_t^i - r_t^{k,i} K_{t-1}^i CPI_t^i = P_t^i Y_t^i (1 - RMC_t)$$

Financial Intermediation

As explained by Schmitt-Grohé and Uribe (2003), the stationarity of a small open economy model is not straightforward and can be ensured by some modeling elements, which are usually not micro-founded. The literature on monetary union model usually borrows the same solutions. In our model, we "micro-found" one of Schmitt-Grohe's proposals (debt elastic spreads) and introduce a simplified interregional financial market.

We assume that there exists an interregional financial market for assets (private or public). On the financial market, intermediaries can borrow money from the central bank of the monetary union to finance public or private credit, and conversely borrow money from agents to deposit it at the central bank. Through financial intermediaries, private (resp. public) agents can borrow or lend money by paying a debt premium ψ (resp. ψ^g). The interest rate for the exchange between the central bank and the financial intermediary is the interest rate set by the central bank. To ensure the orthogonality of financial intermediaries with respect to the rest of the monetary union, we assume that they work in perfectly competitive market and that their unique cost is the refinancing cost vis-a -vis the central bank. Assuming so generates no wage payment or capital and intermediate consumption purchases in this branch of activity hence no transfer between the real economy within the monetary union and financial operators located outside this union. Therefore, developments on the financial market do not affect the rest of the system.

Concretely, if households or the government in region *i* are net borrowers (i.e. $FA_t^i \leq 0$ or $A_t^i \leq 0$), this agent has to pay an interest premium on his debt amounting to $|\psi(fa_t^i)|$, $|\psi^g(pa_t^i)|$. When the agent is net lender, returns are reduced by this same spread captured by the intermediary. As for good producing firms, we assume that financial intermediaries are owned by *Ricardian* households and their profits (i.e. the sum of collected spreads) are paid lump-sum to *Ricardian* households $(FD_t^{(1,2)})$.

In addition, we assume that at each period, the financial intermediaries clear their position towards the central bank:

$$FA_t^1 + FA_t^2 + PA_t^1 + PA_t^2 = 0$$

This condition assumes that public debt is being held entirely by households within the union. This constraint ensures that the model satisfies the Walras law, and that the steady state is stable and the solution to the linearized model is unique. We however allow for a transitory discrepancy in this condition to enable us to introduce debt shocks.

Fiscal Authorities

The endogenous government behavior is modeled through a budget rule so that each regional government adjusts its public expenditures in order to ensure debt is on sustainable path. Namely, we set the following rule:

$$G_t^i - \bar{G}^i = \rho_g \left(p a_{t-1}^i - \overline{p a}^i \right)$$

where G_t^i denotes the level of public expenditures in region *i* and $pa_t^i = PA_t^i P_t^i \overline{Y}^i$ the public debt to GDP ratio. \overline{G} and \overline{pa} denotes the steady level of each variable.

Those expenditures are financed through constant and discretionarily chosen tax rates over consumption and labor as well as debt. In addition, governments also distribute constant lump-sum transfers to both types of households. All in all, the government budget constrain is as follows:

$$PA_{t}^{i} = \left(R_{t-1} - \psi^{g}\left(\frac{PA_{t-1}^{i}}{P_{t-1}^{i}\bar{Y}^{i}}\right)\right)PA_{t-1}^{i} + \nu^{w,i}W_{t}^{i}L_{t}^{i} + \nu^{c,i}CPI_{t}^{i}(C_{t}^{i} + I_{t}^{i}) - P_{t}^{i}G_{t}^{i} - \Phi^{i}$$

where PA_t^i denotes the nominal public assets of region *i* at the end of period *t*. The budget balance decomposes along the value-added tax (v^c) base $CPI_t^i(C_t^i + I_t^i)$ including private consumption and investment valued at the consumption price, and the labor tax (v^w) base $W_t^i L_t^i$. Public consumption is denoted G_t and Φ^i nominal transfers to households. In addition, R_t denotes the nominal interest paid on financial assets reduced/augmented by negligible transaction spreads $\psi^g \left(\frac{PA_{t-1}^i}{P_{t-1}^i \overline{Y}^i}\right)$.

Central bank

The central bank sets the nominal interest rate R_t common to both regions through a Taylor rule (Taylor, 1993), where it reacts to current inflation of the consumption price index.

$$R_{t} = (R_{t-1})^{\rho} (R^{*})^{1-\rho} \left(\frac{\prod_{t}^{union, VAT}}{\Pi^{*}}\right)^{r_{\pi}(1-\rho)} + \varepsilon_{t}^{M}$$

where $\Pi_t^{union,VAT}$ is the consumption-weighted VAT-included CPI inflation in the monetary union. R^* is the interest-rate target of the central bank and Π^* its exogenous inflation target. r_{π} is the Taylor rule weights assigned to inflation, ρ is the interest-smoothing parameter. ε_t^M corresponds to a transitory monetary shock.

Debt shock vis-a-vis the rest of the world

As explained in Campagne and Poissonnier (2016), all financial assets are assumed to be held by households within the monetary union, so that no financial nor trade transactions exist with the rest of the world. As such, at each period, the financial intermediaries clear their position towards the central bank:

$$FA_t^1 + FA_t^2 + PA_t^1 + PA_t^2 = 0$$

where FA_t^i is the level of private financial assets in the region *i*.

However, in the present paper, this assumption is highly restrictive when trying to model shocks to public debt to GDP ratios. Indeed, it implies that in order to match increases in public debt ratios observed in the euro area post-crisis, households' assets would mechanically have to increase. For post-crisis simulation purposes, this is however problematic as it would imply a large positive wealth effect for households that do not reflect post-crisis data. The budget constraint 3.2 is therefore modified to introduce a transitory real debt to GDP shock $\varepsilon_t^{d,i}$:

$$pa_{t}^{i} = \left(R_{t-1} - \psi^{g}(pa_{t-1}^{i})\right) \frac{pa_{t-1}^{i}}{(1+g)\Pi_{t}^{i}} + v_{t}^{w,i}w_{t}^{i}l_{t}^{i} + v_{t}^{k,i}r_{t}^{k,i}cpi_{t}^{i}k_{t-1}^{i} + v_{t}^{c,i}cpi_{t}^{i}(c_{t}^{i}+i_{t}^{i}) + v_{t}^{D,i}d_{t}^{i} + v_{t}^{FD,i}fd_{t}^{i} - g_{t}^{i} - \phi_{t}^{i} + \varepsilon_{t}^{d,i}$$

where $x_t^i = X_t^i P_t^i$ for X = W or CPI, $y_t^i = Y_t^i \overline{Y}^i$ for Y = L, C, I or $G, z_t^i = Z_t^i P_t^i \overline{Y}^i$ for Z = D, FD or Φ and $k_{t-1}^i = K_{t-1}^i P_t^i$.

The $\varepsilon_t^{d,i}$ is assumed with no compensation to the household, that is:

$$FA_{t}^{1} + FA_{t}^{2} + PA_{t}^{1} + PA_{t}^{2} = P_{t}^{1}\bar{Y}^{1}\varepsilon_{t}^{d,1} + P_{t}^{2}\bar{Y}^{2}\varepsilon_{t}^{d,2}$$

The $\varepsilon_t^{d,1}$ and $\varepsilon_t^{d,2}$ shocks can be seen as real debt shocks vis-a-vis a third party (the rest of the world) able to trade in assets with the euro area agents and assumed following a budget rule similar to the euro area ones in order to stabilize its external debt with respect to the euro area.

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