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Abstract

Insee has launched a pilot experiment which aims at introducing scanner data in the French CPI. Started in 2010 with a small set of companies and a small number of industrial food products, the experiment has now reached a larger scale with a daily transmission of data covering 30% of the market. This experiment gives Insee the occasion to review the quality adjustments in the French CPI. Thus, Insee has chosen a strategy of analysis that is mainly based on the same principle as the one applied for the rest of the French CPI: the sample is drawn yearly in the universe of the products in order to reach a certain level of accuracy in the resulting CPI; a two-steps computation is made : the first step consists in computing micro-aggregates while dealing with possible substitutions that occur at the micro-level by the use of adequate price index formulae and the second step consists in the traditional Laspeyres aggregation. The product is followed until it disappears. It is then replaced by a new product after a quality adjustment.

The paper deals with the quality adjustment applied in scanner data. Besides prices and quantities associated to EAN (barcode), each day, and each shop, Insee has bought a database containing descriptive variables of each sold EAN. This information makes it possible to choose in a proper way, replacement products based on a kind of distance between products. It also makes it possible to estimate in an objective way, quality differences with respect to descriptive variables. We compare, on a subset of 13 product families (food and manufactured goods), the results obtained through different techniques of quality adjustment and discuss the advantages and disadvantages of the various techniques with respect to the numerical differences we get. We find on yogurts, chocolate bars, soft cheese, toilet tissue and fruit juice families that quality adjustment is necessary since a quality-adjusted price index differs significantly from a non quality-adjusted index. Furthermore, all quality correction methods tested in the paper lead to statistically-identical price indices. Nevertheless, some systematic differences exist between classical methods of quality adjustment tested here. These differences are negligible as the accuracy required for the index is not too high (up to 20 times the level reached for an index based on traditional collection). But if the required accuracy is higher, then the differences between the quality adjustment methods may become problematic, even for food product as tested in this paper.

JEL Codes : E31 ; C8 ; D1.
Key-words : Price indices, quality adjustment, scanner data.

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1 Introduction

The main French retail chains have used, for quite a long time, centralized databases for management of stocks. In these databases, the products are identified with their EAN (European Article Number e.g. the barcode). In addition, since the beginning of 2000's, these data are collected by two private companies which, in agreement with the retailers, make market studies with these data. The interest of these data for CPI computation is clear since it gives an exhaustive picture of consumption (daily quantities and prices in all possible shops). A few European countries currently compute their CPI with the use of scanner data and many countries have started to study the introduction of these data in CPI. The French national statistical institute (INSEE) has launched in 2010 a pilot project in order to get some insights into the suitability of these data for CPI purposes. In particular, there are two key issues Insee wants to examine:

1. The first one is the notion of product: in the CPI, the elementary product is identified physically from its characteristics observable to the consumer. In scanner data, the product is identified through its EAN code. And we know that the same product (in the classical CPI sense – see § "Notion of product and selection of replacements" – page 5) might have different EANs. It is the case for some discount packagings. Generally, in that case, the consumer may still identify the difference. But there are cases where a perfectly unique product has two or more different EANs. For yogurts for example, if the product is produced in one plant or another, it might have different EANs. And we found cases in our database where the same physical product is sold with different EANs at the same time in the same shop. This happens because this shop, for this product, is supplied by two different factories. Then we see that the concept of product in the CPI meaning is not confounded with concept implicitly described by the EAN.

2. The second issue is to adapt the usual data process which is more or less suitable for a certain volume of information to a volume of data that has nothing in common with the current one. For example, in the French CPI, about 30 000 price observations are done each year for yogurts, while there are 2 000 000 price observations for yogurts in the scanner database we use for this paper, which only covers 25% of the main French retail chains.

Of course, the data processing we may think of is intimately related to the information we have. The data we use in this paper are presented in section 2. Without going into details now, we may say that in addition to daily sales (price and quantities), we have a full set of variables describing each EAN-identified product. This allows to make mass treatment of these data too.

This paper does not deal specifically with the first issue above: we use a sample of the data and we create time series of products through a reasonable algorithm[1] to select replacing article when an article that was followed is missing in the shop where it was sold (– see § "Notion of product and selection of replacements" – page 5). The size of the sample is about 20 times larger than the sample followed in the current CPI for the same families of product. Insofar we make a sample selection, this approach is replicable on various samples and allows us to estimate error bars by bootstrap for our indices estimates and then to compare various methods of index computation.

This paper then mainly deals with the second issue presented above: insofar we may think of mass treatment of the main price information together with meta-data on the products, we may review the quality adjustment method. Indeed, for food goods, Insee use the so-called method of the bridged overlap with a real price increase to monitor quality adjustment (Armknecht, Moulton & Stewart 1994, Armknecht & Moulton 1995, Triplett 2006) when a followed product is missing. This is the best that can be done under the set of information

[1] Actually, we use two different algorithms of replacement: a central one and an alternative one designed to test the robustness of conclusions drawn from the central algorithm.

we collect with the traditional price collection. In particular, the price of a possible replacing good is never observed together with the price of the replaced good at the same time period since the choice of replacing product is made once the previous one has disappeared. And the main part of the observed characteristics aims at identifying the product in the shop rather than comparing the products. The situation with scanner data is different: when we think of replacing a product by another one, it is a very easy task to look back and measure the price of the two products when both were sold (if such a situation has existed) and insofar we have a full set of EAN characteristics, we can *compare* the two products in terms of characteristics.

The paper then focuses on this issue: we compare with scanner data different strategies of quality adjustment and discuss the advantages and disadvantages of each of them.

The paper is divided into three parts: first we present the data and the basic algorithm for CPI computation used here; then we go into the quality adjustment method employed in the paper and we finally present the results we got.

2 From the data to a CPI

At the beginning of the project, at the end of 2010, Insee initiated a discussion with the main French retailers chains in order to get access to a sample of scanner data. Some of these retail chains have accepted to give access to their scanner data (about 25% of the potential "market"). Then Insee bought to the Symphony IRI Group (SIG) a test-sample of 3 years of scanner data.
This sample covers the weekly sales data (quantities, expenditures and prices of the sales) for 17 families of products for 1 000 hypermarkets and supermarkets during three years (2007 to 2009).
The sample contains 141 400 000 observations (45.6 millions in 2007, 47.0 millions in 2008 and 48.8 millions in 2009). An observation corresponds to the sales (quantities and price) of a barcode in a store during one week. For the present paper we studied the following families of products: yogurts, chocolate bars, blue cheese, soft cheese with a washed rind, soft cheese with a bloomy rind, other cheese, chicken egg, frozen pizzas, toilet tissue, fruit juice, ground coffee with caffeine, ground coffee without caffeine, ground coffee in pods. In average and for example, 224 different EANs of yogurts were sold per supermarket in December 2008 and 201 of Chocolate bars.
The basic information contained in the files for a given triplet [EAN, shop, week] is the number of products sold and the price (eventually averaged when different prices were applied for an elementary cell).

Two additional files were bought by Insee: the first one describes each barcode through a set of variables such as brand, perfume, packaging... (see table 1 for an overview of the file contents for yogurts as an example of product). The second one describes the shops (company, city, area...).

Table 1: Variables describing the yogurt family

Variables	Categories	Main values
IRI Family	1	5701
Family description	1	yogurts
Brand	112	Activia (17%), Panier (9%), U (9%), Taillefine (8%), Velouté (6%), ...
Bonus 1	13	12/4 (30%), 8CO (27%), 8/4B (16%), 4/2B (7%)
Bonus 2	0	-
Location	1	Refrigerated cabinet
Type of product	2	yogurts (99,97%), Batch of yogurts (0,03%)
Packing	6	Plastic pot (89%), Glass pot (8%), Cardboard pot (2%), Stoneware (1%), Bucket (0%), Bi-comp pot
Perfume variety	202	Natural (22%), Assorted fruit (12%), Vanilla (8%), Strawberry (6%), Red fruit (4%)
Fair Trade Information	0	-
Active ingredient	3	Bifidus (94%), Anti ch (6%), Omega 6 (0,2%)
Ethnic info	2	Standard (99,99%), Halal (0,01%)
Promoting information	7	Shock price (71%), Special price (14%), Special Offer (8%), Eco Pack (5%), Offers eco (2%)
Biological information	3	Non bio (94%), Bio (5%), Bio AB (1%)
Content of milk	4	Full-cream milk (69%), Skimmed milk (25%), Plant (6%), Semi-skimmed milk (0,5%)
Additives	32	Pieces of fruits (66%), Pulp (17%), Fruits in the background (8%), Fruit bed (2%)
Sugar content	3	Sweetened (78%), Unsweetened (20%), Sugar of cane (3%)
Process	10	Standard (39%), Farm (32%), Stirred (17%), Bilayer (6%), Creamy (3%)
Fat content in %	7	0% MG (98,5%), 6,5% MG (0,7%), 2,9% MG (0,5%)
Fat content	2	Regular (75%), Reduced (25%)
Type of yogurt	5	Fruit yogurt (56%), Natural yogurt (22%), Flavoured yogurt (22%)
Total volume	42	500gr (46%), 1000gr (14%), 1500gr (10%), 2000gr (8%), 400gr (6%), 300gr (5%)
Number of units in the pack	8	4ct (54%), 8ct (14%), 2ct (10%), 12ct (10%), 16ct (8%), 1ct (2%), 6ct (2%), 3ct (0%)
Number of bags per pack	1	1ct
Volume per unit	23	125gr (79%), 150gr (7%), 100gr (7%), 115gr (2%), 140gr (1%), 120gr (1%), 135gr (1%)

Note: all the variable are discrete ones. Column "Categories" indicates the number of different possible values and the column "Main values" the most frequent values with the corresponding percentage frequency in parenthesis.

Notion of product and selection of replacements The statistical unit followed in the price index cannot be coincident with the article identified by its EAN insofar the same product (in the sense of the traditional CPI or in the view of the consumer) might have different EAN's. For example, discounts for bulk are very common and to a certain extent of quantities, considered, in the current CPI, as the same product as the original one. The price of the product is the unit price of a product defined with a reference volume of content. When the volume increases in case of discounts for bulk, then the unit price per volume decreases, everything else being equal.

Therefore, the product in the sense of CPI is related to the EAN with a $1 \leftrightarrow N$ relationship at each period of time, which is called here "an equivalence class". The equivalence class contains all the EAN's sold in a shop related to the same product (more or less a main EAN and additional EAN's corresponding to the same product with the same characteristics except from the volume of material that might differ). Most of the time, an equivalence class in a given shop contains only one EAN[a]. When it contains more than one EAN, it contains generally the one of the generic product as well as EAN's associated to discount packagings.

The notion of product is also related to time support : when the equivalence class disappears, a successor must be found to continue the time series of prices. In this vision, the product is a permanent concept. When, at one time, its equivalence class vanishes, a new one is chosen as the representative of the product. A replacement is made and the price time series continues with, when necessary, a quality adjustment.

In this paper, we use two algorithms to select the replacing equivalence class.

I. [**Central algorithm**] In the first one, we select the replacing equivalence class in the same variety of product, in the same shop, in the same brand and in the same range of volume. If there is no candidate, then we relax the brand criteria and try to find within the same variety and the same outlet. If there is still no candidate, we try to find within the same variety, same city and same brand. Again, if no candidate is found, we try to find within the same variety and the same city. Next step, the same variety and same brand (with no geographic criteria). And last, we try to find in the same variety.

   At each selection step, the selected equivalence class must be observed during months $M-1$ and $M-2$ as well as during the current month M. If at any previous step, there are more than one candidate, the selected one is the one whose price is the closest to the price of the replaced good in month $M-1$. If there are still a few goods whose price was equal to the previous price of the replaced good during month $M-1$, then the selected one is that whose sold quantities were the closest to the sold quantities of the replaced good (at the same considered geographic cell).

II. [**Alternative algorithm**] The alternative algorithm is devoted to test the robustness of the conclusions drawn from the central processing (algorithm I above). In this second algorithm, we first try to find a candidate within the same variety and the same outlet. Second step (if there is no candidate at the first step), we try to find within the same variety and the same city. Last step, in the same variety (regardless geography). This last algorithm is only processed as an alternative test on yogurts and chocolate bars.

   The first stage where there is at least one candidate, if there are more than one, the final one is selected randomly (uniform law).

In the paper, unless otherwise stated, the algorithm used is the number I presented above.

[a] There are 1.3 EAN on average per equivalence class for yogurts in a given instant and 1.2 for chocolate bar

In this data, the basic unit of observation is the EAN barcode crossed with the shop and the week: a mean price and a cumulative quantity sold is given for this unit. It may be followed in time, exactly as the goods are followed in the current CPI. When the product disappears, it must be replaced by another good, close to the first one, exactly as is done currently in the CPI. When a product disappears, we select another one according to proximity. The proximity is built on a subset of characteristics that we consider as the most important ones

(for example, variety[2] of product, outlet, brand, volume range) : the idea is to find a replacing product that has the same characteristics.
Two algorithms are used in this paper for replacements: a central one and an alternative one. The details of the algorithms are given in the frame "Notion of product and selection of replacements" – page 5. The type of replacement is classified into a 6-levels scale, depending on the step of the algorithm where the replacing product was found: from 1 (best type of replacement) to 6 (worst type of replacement). The results are presented in table 2 for the central algorithm and for a few subset of studied families of products.

Table 2: Statistics about the type of replacements

Type	Criteria	yogurts	chocolate bars	blue cheese	chicken eggs	gr. coffee with caf.
1	Same variety, same outlet, same brand	73%	55,7%	58,0%	16,9%	33,8%
2	Same variety, same outlet	27%	44,3%	42,0%	80,2%	66,2%
3	Same variety, same city, same brand	0%	0%	0%	2,8%	0%
4	Same variety, same city	0%	0%	0%	0%	0%
5	Same variety, same brand	0%	0%	0%	0%	0%
6	Same variety	0%	0%	0%	0,1%	0%
Total		*100%*	*100%*	*100%*	*100%*	*100%*

Note: we only give the results for some of the families of products, aiming at presenting the diversity of situations.

After this process, we have continuous series of products (crossing a given EAN and a given shop) with replacement events when necessary. A sample is drawn in these series for products available in November and December 2008. This sample corresponds to a sampling rate[3] of about 2% with a probability of inclusion proportional to the sales of these two months.
A price index is then computed for the whole concerned family over 2009: first an elementary index is computed for the cell defined by the considered outlet and the considered variety of products[4]. Then, this elementary index is aggregated up to the family level through a Laspeyres-type aggregation process[5].

In order to compute error bars over the resulting indexes, a certain number of simulations are drawn for each family of products. This number has been set through a common fixed point criteria applied over the whole set of families (see section 4).

[2] This notion of variety already exists in the current CPI. It corresponds to the ultimate division of products, without being a partition of the consumption. It is defined in a very detailed manner and is viewed as representative (in the statistical meaning) of the sub-part of consumption that it is supposed to represent. For example, for yogurts, there are 6 varieties of products in the current CPI and they cover about half of the family they are supposed to represent. It means that in the CPI, we suppose that these 6 varieties taken as a whole have the same price dynamics as the whole family of yogurts. Therefore, this varieties cover in general well sold and well followed products. The 6 current CPI varieties of yogurts have been identified in the scanner dataset through adequate filters applied on the variables describing EANs characteristics. The same has been done for chocolate bars. The computation is made over the CPI varieties.

[3] The sampling rate is an a priori in this paper. In the choice, we balanced the wish to increase the size of the current CPI sample (multiplied by 20 with the chosen sampling rate of 2%) and the wish to maintain a reasonable size of the sample that allows to keep an eye on replacement automatisms.

[4] The micro index is computed with the use of a Jevons formula. Since the inclusion probability of the sample is proportional to the sales observed in November and December 2008, the micro-index computed here is an estimate of a geometric Laspeyres weighted by the sales of these two months. This approach is a bit different from that applied in the current CPI where the elementary cell is the city, and not the shop. This choice is made in the context of this paper.

[5] Again, since the inclusion probability is proportional to the November and December 2008 sales, any aggregation using an arithmetic mean formula is an estimate of the expense-weighted Laspeyres aggregation formula.

3 Tested quality adjustment methods

Some papers have already identified the potential of scanner data in improving the practical implementation of quality adjustment methods (e.g. Ahnert & Kenny (2004),Silver & Heravi (2000),Silver & Heravi (2005)). Insee will, most probably, introduce scanner data in the field of food products first (except fruits and vegetables). Even if quality adjustment are known to be mainly significant in other sectors of consumption, the availability of scanner data together with precise description of the products invites us to have a look at this issue for food products. This paper aims at reviewing the possible techniques and their practical implementation in the French CPI in the case of food products. This section presents the various algorithms of quality adjustment used in this paper. The terminology used here is the one from the (Center of Excellence Network - CENEX 2008). For an comprehensive view of quality adjustment techniques, see (ILO, IMF, OECD, UNECE, Eurostat & The World Bank 2004).

In this section, we will consider a series of product i and we assume that (new) equivalent class (i.e. good – see § "Notion of product and selection of replacements" – page 5) N replaces the good O (old) at month m for this series. $P^O_{i,m-1}$ is the price of good O for series i at month $m-1$ and $P^N_{i,m}$ is the price of good N for series i at month m. Prices $P^O_{i,m-1}$ and $P^N_{i,m}$ are both observed. The quality adjustment, in the following formulas, is assumed to be made on the price of good O at month $m-1$. Thus we have a price $\tilde{P}^O_{m-1}$ that is supposed to be the price of the series at month $m-1$ such as the level of quality is the same as the new good one. Therefore, the monthly variation of the series i contributing to the index of month m will be:

$$I_{i,m} = \frac{P^N_{i,m}}{\tilde{P}^O_{i,m-1}} \quad (1)$$

We will also define the J coefficient as the inverse ratio of $I_{i,m}$ to the value of the index if the replacement was ignored ($I^0_{i,m} = P^N_{i,m}/P^O_{i,m}$), or equivalently, if no quality adjustment was made:

$$J_{i,m-1} = \frac{I^0_{i,m}}{I_{i,m}} = \frac{\tilde{P}^O_{i,m-1}}{P^O_{i,m-1}} \quad (2)$$

Written like that, $J_{i,m-1}$ appears as the ratio of the computed price of good O with the quality level of good N and the price of good O with its own level of quality. In other words, $J_{i,m-1}$ is the measure of the value that the consumer makes of differences in characteristics of the goods at month $m-1$ (see appendix A for an economic approach of this variable). If $J_{i,m-1} > 1$, the quality of the new good is higher, and vice-versa. There are five different ways of computing $\tilde{P}^O_{m-1}$ (sections 3.1 to 3.5).

3.1 Equivalent products or no quality adjustment[6] (1)

This is the basic case where the goods are supposed to be directly comparable in terms of quality. Then

$$\tilde{P}^O_{m-1} = P^O_{m-1} \quad (3)$$

In this case,

$$J_{i,m-1} = 1 \quad (4)$$

3.2 The link-to-show-no-price change (2)

The technique consists in saying that the full price difference between the old good at month $m-1$ and the new one at month m is due to quality difference. Therefore

$$\tilde{P}^O_{i,m-1} = P^N_{i,m} \quad (5)$$

[6] The numbers in parenthesis indicate the method. They are recalled in the tables of results.

In this case,

$$J_{i,m-1} = \frac{P^N_{i,m}}{P^O_{i,m-1}} \tag{6}$$

3.3 Bridged overlap with real price increase (3)

A classical criticism of the last link-to-show-no-price change method is that, in case of replacement, the contribution of the series to the index is always null, even when there is a significant price change for observed products. In order to reduce this criticism, the Bridged overlap with real price increase method has been developed. It ensures that the contribution of series i is identical to the one obtained for the products really observed for the couple of months $m-1$ and m.

Let us define $\frac{1}{n}\sum_{j=1}^{n}\frac{P_{j,m}}{P_{j,m-1}}$ the average monthly evolution for the n observed products (same categories of products, for example, in the current CPI, same variety and same city). The idea is almost the same as that of previous section (all the difference in price is a quality effect) but we assume that since the quality is identical, the monthly evolution of series i should be identical to the observed monthly price change. Then

$$\tilde{P}^O_{i,m-1} = \frac{P^N_{i,m}}{\frac{1}{n}\sum_{j=1}^{n}\frac{P_{j,m}}{P_{j,m-1}}} \tag{7}$$

and

$$J_{i,m-1} = \frac{P^N_{i,m}}{P^O_{i,m-1}} \times \frac{1}{\frac{1}{n}\sum_{j=1}^{n}\frac{P_{j,m}}{P_{j,m-1}}} \tag{8}$$

3.4 1 or 2 months Overlapping

In this case we assume that the new good N and the old one O are sold together at the same time, during month $m-1$ (1 month overlapping) or month $m-2$ (2 months overlapping). And we assume that the value of the difference in price observed during month $m-1$ or $m-2$ is the value of the difference in characteristics the consumer makes. An important point is that this method is easily applicable with scanner data and much more complicated with traditional price collection. Therefore, we have:

- **1-month Overlapping (4)**:

  $$\tilde{P}^O_{i,m-1} = P^N_{i,m-1} \tag{9}$$

  and

  $$J_{i,m-1} = \frac{P^N_{i,m-1}}{P^O_{i,m-1}} \tag{10}$$

  The quality shift is exactly equal to the price ratio of the new good and the old one at month $m-1$.

- **2-months Overlapping (5)**:

  $$\tilde{P}^O_{i,m-1} = \frac{P^O_{i,m-1}}{P^O_{i,m-2}} P^N_{i,m-2} \tag{11}$$

  and

  $$J_{i,m-1} = \frac{P^N_{i,m-2}}{P^O_{i,m-2}} \tag{12}$$

  The quality shift is exactly equal to the price ratio of the new good and the old one at month $m-2$. The possible advantage of 2-months Overlapping with respect to 1-month is that it may be less sensitive to typical marketing strategies for end-of-life products whose price may be reduced drastically just at the end.

3.5 Hedonic models (6)

In this case, a Hedonic model is estimated through the econometric estimation of a model linking the logarithm of the price to the characteristics of the products. Let us write $\hat{P}^O_{i,m-1}$ the estimated Hedonic price of the old good and $\hat{P}^N_{i,m-1}$ for the new one. Then

$$\tilde{P}^O_{i,m-1} = \frac{P^O_{i,m-1}}{\hat{P}^O_{i,m-1}} \hat{P}^N_{i,m-1} \tag{13}$$

and

$$J_{i,m-1} = \frac{\hat{P}^N_{i,m-1}}{\hat{P}^O_{i,m-1}} \tag{14}$$

Overlapping and Hedonic techniques are both compatible with the usual consumer theory. We can show (see appendix A) that it is possible, in order to realize a constant utility price index with goods of different levels of quality, to correct the prices for the difference in characteristics perceived by the consumer. In that case, the prices themselves might reveal the value the consumer makes on differences in characteristics, provided that a model linking price and characteristics is estimable or that the compared products are both present on the market at the same time. Of course the key issue is that in one way or another, an equilibrium is reached on the market at each period and the difference in price equilibrates the difference in tastes. This last assumption is obviously restrictive but it is useful to keep this model in mind since all constant quality price corrections rely more or less on this idea.

4 Results

All the methods presented above have been tested on two families of products: yogurts (section 4.1) and chocolate bars (section 4.2). Results are then generalized for the rest of families of products (section 4.3). The reference method for quality adjustment is the Hedonic one (estimation based on product characteristics). A Hedonic model has been built for each family. Estimation and results of the Hedonic model estimations for yogurts and chocolate bars, as examples, are presented in appendix B.

100 samples are selected in order to simulate the results for yogurt and 200 for chocolate bars. For all other families, the number of selected samples is set to 30. The price change is computed between December 2008 and December 2009 for every samples. Elementary indices are Jevons indices at [variety] x [outlet] level. The index for a family is the result of a Laspeyres aggregation of micro-indices (weighted by the elementary expenditure on the base period – i.e. nov.-dec. 2008).

In this part of the text, a quality adjustment is applied whatever the type[7] of replacement is. In annex C, we study, in the case of yogurts and chocolate bars, the consequence of applying a mixed strategy: no quality adjustment (i.e. equivalent replacements) for type 1 replacements (see table 2) and quality adjustment for type 2 and higher types of replacements. Of course, the difference of indices between the two ways of adjusting quality effects in case of type-1 replacements should not be understood as inconsistencies but rather like a possible failure of the fact that type-1 replacements are *true* equivalent replacements (i.e. a priori assumed to be free of any quality effect).

In order to test the robustness of the conclusions that can be drawn from the index computed with the central algorithm of replacements, a second computation is made with the alternative algorithm (see the frame "Notion of product and selection of replacements" – page 5). Indeed, if one believes in the meaning of quality adjustment (see appendix A), a change in the choice of replacing goods should be neutral to the index. It is what is shown in the last parts of sections 4.1 and 4.2, as well as in section 4.3, for the rest of families of products.

[7] See table 2 for the meaning of "type of replacements".

4.1 Simulations on yogurt family

The first results concern the yogurt family (tables 3 to 6). First of all, the monthly price evolution for all the couple [EAN] x [month] between December 2008 and December 2009 is negative: on the period, the price decrease by 0.013% with a large standard deviation equal to 8.2% (table 3).

Table 3: *[yogurts]* Price evolution between month m and m-1 for not missing products

Number of units (1)	Average	Std-Dev	Percentile 5%	Median	Percentile 95%
4 267 697	-0.013%	8.2%	-12.7%	0.0%	12.7%

Note: (1) the number of unit corresponds to the set of all non-replaced products in the union of all selected samples over 12 months.

Replacement are not very frequent: they represent about 1% of the monthly price evolutions taken into account to compute the simulated 100-samples price indices. In case of replacement, the magnitude of the quality correction may greatly differ from one technique to the others, for a given product (table 4). But a test for a difference in mean does not reject equality except for overlapping methods which seem to slightly overestimate quality correction with respect to Hedonic's. But this slight bias is, at the end, diluted in the index noise[8] and does not generate a difference in the indices (see hereafter).

Table 4: *[yogurts]* Difference of estimated quality factors by Hedonic's and the orther techniques

Type of quality adjustment	Difference in Mean with respect to Hedonic's	Distribution of the difference		
		Perc. 5%	Median	Perc. 95%
(2) Link-to-show-no-price-change	-0.006 $[-0.017\ ,\ 0.003]$	-0.22	0.00	0.17
(3) Bridged overlap with real price increase	-0.010 $[-0.020\ ,\ -0.001]$	-0.22	0.00	0.16
(4) Last month overlapping	-0.016 $[-0.024\ ,\ -0.009]$	-0.19	-0.01	0.12
(5) Penultimate month overlapping	-0.008 $[-0.016\ ,\ -0.001]$	-0.16	0.00	0.13

Note: The considered statistic is the difference between the elementary J coefficient (Eq. 2) computed with the Hedonic model and the one computed with the considered method. The column "Difference in mean" corresponds to the average difference over a sample. The associated 95% confidence interval is computed from the distribution of the sample average values observed from the simulated 100 samples.

The three columns on the right correspond to the distribution of the 42703 difference in J coefficients, computed with the full set of the elementary cases of replacements encountered in the 100 samples.

When the mean is negative, the quality correction is larger for the considered method than for Hedonic's and price inflation is therefore lower. The column "Difference in mean" shows the possible systematic difference between Hedonic's J coefficient and that of the considered method. When 0 is not contained in the 95% confidence interval, the J coefficient differs significantly in mean with Hedonic's.

We show in table 5 the results for the distribution of the yearly index evolution for yogurts depending on the applied technique for quality adjustment. This distribution is computed from the elementary computation of 100 indices based on the same number of selected samples. Except from the no-quality correction price index, all quality corrected indices are statistically consistent[9]. The link-to-show-no-price-change is very close to the index based on a Hedonic model. Bridged overlap model and overlapping are a little bit further but

[8] Because the frequency of the replacements is small and the bias is small too.

[9] statistically consistent means that we cannot reject the null hypothesis of an equality test between any couple of indices.

still statistically consistent. Interestingly, there is a significant difference (at the 95% level) between the index computed with no quality correction and the one computed with the Hedonic model for quality adjustment. This shows that quality adjustment has a significant impact on price indices even for yogurts where we may think, at first glance, that quality issues are of minor importance.
The highest yearly evolution is obtained with Hedonic model for quality adjustment, consistently with table 4 observations.

Table 5: *[yogurts]* Price evolution between December 2008 and December 2009

Type of quality adjustment	Mean annual increase
(1) No quality correction	-4.14% $[-4.5\%\ ,\ -3.8\%]$
(2) Link-to-show-no-price-change	-3.55% $[-3.9\%\ ,\ -3.3\%]$
(3) Bridged overlap with real price increase	-3.59% $[-3.9\%\ ,\ -3.3\%]$
(4) Last month overlapping	-3.71% $[-4.0\%\ ,\ -3.4\%]$
(5) Penultimate month overlapping	-3.60% $[-3.9\%\ ,\ -3.3\%]$
(6) Hedonic model	-3.52% $[-3.8\%\ ,\ -3.2\%]$

Note: computation made over 100 annual simulations. The intervals in brackets correspond to 95% confidence intervals. Central algorithm of replacement used.

Table 6 shows the results obtained while the alternative algorithm for selecting replacing goods is used. The comparison with table 5 shows that the estimated price index, while a quality correction is applied, is statistically consistent even if the not-quality-corrected index is significantly different from its counterpart obtained with the central algorithm of replacement. It shows that the alternative algorithm used to select replacement goods does not select the same replacing products up to a point that not-quality-corrected indices are different. This was actually what we aimed at doing in this exercise of comparison. And the fact that quality corrected indices are consistent shows, in some way, that the concept of quality adjusted price index is not sensitive to the algorithm selecting the replacing goods. This is a highly desirable property because it suggests, consistently with the Consumer theory (annex A), that the measure of quality computed with the various techniques used for adjusting the quality factor actually measures the same theoretical concept.
This result has the practical consequence that the choice of the replacing product is, to some extent, of minor importance providing that the method used for quality correction is unbiased according to the Consumer theory (see annex A).

4.2 Simulations on Chocolate bars family

In this section, we test the methods of quality adjustment on the family of chocolate bars. Table 7 shows the average monthly price evolution observed over the whole sample of products between December 2008 and December 2009. Prices are increasing a bit and the distribution is much narrower than in the case of yogurts: the 95% interval is [-5.5%; +5.3%] for chocolate bars; it was [-12.7%; +12.7%] for yogurts.
Replacement are again not very frequent but relatively more frequent than in the case of yogurts: they represent about 3% of the monthly price evolution taken into account to compute the simulated 200-samples price indices. In case of replacement, the magnitude of quality correction may again greatly differ from one technique to the others, for a given product (table 8). But a test for a difference in mean does not reject equality except for the penultimate month overlapping method which seems to slightly overestimates quality

Table 6: *[yogurts]* Price evolution between December 2008 and December 2009 for alternative replacement algorithm

Type of quality adjustment	Mean annual increase
(1) No quality correction	-3.17% $[-3.6\% \ , \ -2.7\%]$
(2) Link-to-show-no-price-change	-3.51% $[-3.8\% \ , \ -3.2\%]$
(3) Bridged overlap with real price increase	-3.56% $[-3.8\% \ , \ -3.2\%]$
(4) Last month overlapping	-3.60% $[-3.9\% \ , \ -3.3\%]$
(5) Penultimate month overlapping	-3.51% $[-3.8\% \ , \ -3.2\%]$
(6) Hedonic model	-3.52% $[-3.8\% \ , \ -3.2\%]$

Note: computation made over 100 annual simulations. The intervals in brackets correspond to 95% confidence intervals. Alternative algorithm of replacement used.

Table 7: *[Chocolate bars]* Price evolution between month m and m-1 for non replaced products

Number of units (1)	Average	Std-Dev	Perc. 5	Perc. 50	Perc. 95
4 807 744	0,054%	4.4%	-5.5%	0.0%	5.3%

Note: (1) the number of units corresponds to the set of all non-replaced products in the union of all selected samples over 12 months.

correction with respect to Hedonic's. But this slight bias is, at the end, diluted in the index noise and does not induce a difference in the indices (see hereafter).

Table 9 shows the results for the distribution of the yearly index evolution for chocolate bars depending on the applied technique for quality adjustment. This distribution is computed from the elementary computation of 200 indices based on the same number of selected samples. The quality-adjusted price indices are very consistent and look quite similar whatever the quality adjustment method is. The yearly evolution seen by the price index where no quality-adjustment is done is significantly different from the yearly evolution of corrected indices. It shows that the quality adjustment is a more important issue for chocolate bars than for yogurts.

Table 10 shows the result of the simulations while the alternative algorithm is used in case of replacing product selection. As for yogurts, the not-quality-corrected price index differs significantly from its counterpart obtained with the central algorithm. It shows that we succeeded in selecting replacing products in a significantly different manner. Then we can identify whether or not the quality corrected indices are still consistent with the ones obtained using the central algorithm.

Comparing tables 9 and 10, the answer is yes: for any quality-adjusted index, the null hypothesis in an pairwise equality test is never rejected.

Finally, the conclusion is the same as in the yogurt case: the index does not seem to be sensitive to the choice of the replacing products, providing that quality correction is unbiased, which is practically the case for the techniques considered in this paper, especially while taking into account the relatively small frequency of replacements in the balance.

Table 8: *[Chocolate bars]* Difference of estimated quality factors by Hedonic's and the orther techniques

Type of quality adjustment	Difference in Mean with respect to Hedonic's	Distribution of the difference		
		Perc. 5%	Median	Perc. 95%
(2) Link-to-show-no-price-change	-0.004 $[-0.012\ ,\ 0.003]$	-0.22	0.00	0.19
(3) Bridged overlap with real price increase	-0.005 $[-0.013\ ,\ 0.002]$	-0.22	0.00	0.19
(4) Last month overlapping	-0.004 $[-0.011\ ,\ 0.003]$	-0.21	0.00	0.19
(5) Penultimate month overlapping	-0.009 $[-0.015\ ,\ -0.004]$	-0.21	0.00	0.17

Note: The considered statistic is the difference between the elementary J coefficient (Eq. 2) computed with the Hedonic model and the one computed with the considered method. The column "Difference in mean" corresponds to the average difference over a sample. The associated 95% confidence interval (in brackets) is computed from the distribution of the sample average values observed from the simulated 200 samples.
The three columns on the right correspond to the distribution of the 145 856 difference in J coefficients, computed with the full set of the elementary cases of replacements encountered in the 200 samples.

When the mean is negative, the quality correction is larger for the considered method than for Hedonic's and price inflation is lower. The column "Difference in mean" shows the possible systematic difference between Hedonic's J coefficient and that of the considered method. When 0 is not contained in the 95% confidence interval, the J coefficient differs significantly in mean with Hedonic's.

Table 9: *[Chocolate bars]* Price evolution between December 2008 and December 2009

Type of quality adjustment	Mean annual increase
(1) No quality correction	1.90% $[1.4\%\ ,\ 2.5\%]$
(2) Link-to-show-no-price-change	-0.23% $[-0.5\%\ ,\ 0.1\%]$
(3) Bridged overlap with real price increase	-0.24% $[-0.6\%\ ,\ 0.1\%]$
(4) Last month overlapping	-0.23% $[-0.5\%\ ,\ 0.1\%]$
(5) Penultimate month overlapping	-0.35% $[-0.7\%\ ,\ 0.0\%]$
(6) Hedonic model	-0.11% $[-0.4\%\ ,\ 0.2\%]$

Note: computation made over 200 annual simulations. The intervals in brackets correspond to 95% confidence intervals. Central algorithm of replacement used.

Table 10: *[Chocolate bars]* Price evolution between December 2008 and December 2009 for alternative replacement algorithm

Type of quality adjustment	Mean annual increase
(1) No quality correction	9.10% [7.5% , 10.6%]
(2) Link-to-show-no-price-change	−0.03% [−0.3% , 0.3%]
(3) Bridged overlap with real price increase	−0.04% [−0.3% , 0.3%]
(4) Last month overlapping	−0.03% [−0.3% , 0.3%]
(5) Penultimate month overlapping	0.01% [−0.3% , 0.3%]
(6) Hedonic model	0.13% [−0.1% , 0.4%]

Note: computation made over 200 annual simulations. The intervals in brackets correspond to 95% confidence intervals. Alternative algorithm of replacement used.

4.3 Generalization for other families of products

The same analysis has been carried out over the rest of the set of data, namely: 4 families of cheese, frozen pizzas, chicken eggs, toilet tissue, fruit juice, 3 types of coffee products. Table 11 shows the results of the central algorithm of replacement for all other families of products apart from yogurts and chocolate bars.

As previously stated on yogurts and chocolate bars, the results of quality-adjusted CPI do not differ from each other and this, for any family of product.

Interestingly, the not-quality corrected index (column (1) of table 11) is significantly different from quality corrected index (columns 2-6 of table 11) in the cases of soft cheese with washed rind, toilet tissue and fruit juice. This demonstrates again, together with the same result obtained on chocolate bars, the importance of this issue of quality adjustment, even for products for which we may think first that quality effect are of second order.

In the case of toilet tissue, as in the case of chocolate bars, a shift in the Hedonic-corrected index is observable with respect to other quality-corrected indices: the confidence intervals of Hedonic-corrected and penultimate month overlapping-corrected indices even do not overlap each other (but is very close to Overlapping).

Except from this case, the apparent bias that was suggested by data in the case of yogurt and chocolate bars (tables 4 and 8) does not appear in any case for other families of products.

5 Conclusion

We have shown the possible use that could be done of scanner data when an additional set of variables describing the EAN is available. Indeed, this additional set makes it possible to follow exactly the concepts of the traditional Laspeyres type index, while improving their realization. For studied food products, while a raw algorithm for selecting replacing goods in case of replacement is used, quality effects are significant, even when the number of replacements is small. In that case, Hedonic method still appears as the reference method, but Bridged overlap with real price increase, Overlapping and even Link-to-show-no-price-change methods do not lead to significantly different price indices. Nevertheless, in case of replacement, the estimated correction coefficient may differ significantly of Hedonic's quality adjustment which shows, even if the consequences are

Table 11: *[Other families of products]* Price evolution between December 2008 and December 2009

Other Families of product	Type of quality adjustment (1)	(2)	(3)	(4)	(5)	(6)
blue cheese	2.67 [1.87 , 3.47]	2.43 [1.74 , 3.12]	2.47 [1.78 , 3.17]	2.41 [1.71 , 3.11]	2.52 [1.90 , 3.14]	1.96 [1.38 , 2.53]
soft cheese with a washed rind	1.45 [0.85 , 2.05]	−0.19 [−0.87 , 0.50]	−0.16 [−0.86 , 0.55]	−0.23 [−0.84 , 0.39]	−0.20 [−0.88 , 0.48]	−0.09 [−0.74 , 0.55]
soft cheese with a bloomy rind	−3.76 [−4.26 , −3.27]	−3.99 [−4.48 , −3.50]	−4.03 [−4.53 , −3.53]	−4.16 [−4.65 , −3.67]	−4.11 [−4.61 , −3.61]	−4.00 [−4.53 , −3.46]
other cheese	0.62 [−2.75 , 4.00]	0.95 [−0.87 , 2.78]	0.93 [−0.96 , 2.82]	0.79 [−1.09 , 2.67]	0.88 [−1.15 , 2.91]	0.99 [−0.82 , 2.80]
chicken egg	−0.58 [−1.05 , −0.10]	−0.76 [−1.09 , −0.43]	−0.78 [−1.11 , −0.45]	−0.82 [−1.14 , −0.51]	−0.81 [−1.15 , −0.46]	−0.80 [−1.19 , −0.40]
frozen pizzas	1.15 [0.16 , 2.13]	0.51 [−0.51 , 1.53]	0.53 [−0.57 , 1.62]	0.71 [−0.48 , 1.89]	1.22 [0.36 , 2.08]	1.02 [−0.01 , 2.04]
toilet tissue	1.53 [0.80 , 2.27]	−0.55 [−1.01 , −0.09]	−0.54 [−1.02 , −0.07]	−0.60 [−1.08 , −0.12]	−0.68 [−1.13 , −0.22]	0.17 [−0.23 , 0.56]
fruit juice	3.31 [2.65 , 3.98]	1.63 [1.13 , 2.12]	1.68 [1.09 , 2.27]	1.76 [1.29 , 2.23]	1.95 [1.47 , 2.43]	1.65 [1.13 , 2.18]
ground coffee with caffeine	3.35 [2.87 , 3.84]	3.03 [2.63 , 3.43]	3.19 [2.76 , 3.61]	3.19 [2.78 , 3.59]	3.19 [2.70 , 3.68]	3.85 [3.29 , 4.42]
ground coffee without caffeine	2.08 [1.19 , 2.97]	1.41 [0.44 , 2.37]	1.48 [0.46 , 2.50]	1.16 [0.31 , 2.01]	1.33 [0.62 , 2.05]	1.54 [0.84 , 2.24]
ground coffee in pods	3.35 [2.29 , 4.41]	2.53 [1.72 , 3.35]	2.60 [1.75 , 3.44]	2.63 [1.89 , 3.36]	2.69 [1.91 , 3.48]	3.54 [2.68 , 4.41]

Note: numerical values expressed in index points (%). Central algorithm used. Computation made over 30 annual simulations. The intervals in brackets correspond to 95% confidence intervals. The heading of columns correspond to the type of quality adjustment applied: (1)=No quality correction; (2)=Link-to-show-no-price-change; (3)=Bridged overlap with real price increase; (4)=Last month overlapping; (5)=Penultimate month overlapping; (6)=Hedonic model.

negligible for the index, that we must keep an eye on this. In particular, if an higher accuracy of the price index is requested, then an increase of the sample size would lead to a less favorable signal to noise ratio and, probably, the systematic difference that exists between the various techniques of quality adjustment would at the end, lead to significantly different price indices. In the context of this paper, the Link-to-show-no-price-change appears as the only unbiased method with respect to Hedonic's at the 95% level (see tables 4 and 8). At the opposite side, the Penultimate month overlapping is the only systematically biased method at the same level of confidence. From a practical point of view, all the methods that rely on the traditional economic model of consumer theory might be considered in scanner data analysis, Overlapping being the simplest one. This interesting result needs to be generalized to other families of products.

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A Quality and Constant Utility Framework

This section follows mainly the classical approach of constant quality indices presented in the framework of the Constant Utility price index proposed by Deaton & Muelbauer (1980).
We assume that the price index consists in following a set of products which covers a set of needs. These products are consumed by a representative consumer who decides on the quantities of products he buys after having optimized a utility function. This utility function operates on the vector of quantities referring to the previous set of products. Let us write u the utility function[10] and $\mathbf{q}$ the vector of quantities of products bought at any period[11] of time.
The problem of consumer is the following: its Marshallian demand is

$$\mathbf{x}_u(\mathbf{p};R) = \underset{\mathbf{q}}{\operatorname{argmax}}\,\{u(\mathbf{q})|\mathbf{p}.\mathbf{q} = R\}$$

and the expense function, dual of the previous one, is:

$$e_u(\mathbf{p},\bar{u}) = \min_{\mathbf{q}}\,\{\mathbf{p}.\mathbf{q}|u(\mathbf{q}) = \bar{u}\}$$

Since the two problems are dual of each other, we have the usual relationship between the two: the expense associated to the optimized basket given by the Marshallian demand is equal to the expense function taken at this optimized basket utility point, the same price vector being taken for the two problems. Formally, this is written:

$$\mathbf{p}.\mathbf{x}_u(\mathbf{p},R) = e_u(\mathbf{p}, u(\mathbf{x}_u(\mathbf{p},R)))$$

In this model, the Utility constant price index, by definition, follows the evolution of the budget the consumer needs to spend in order to keep its utility constant between the two periods of comparison 0 and t:

$$I_t^0 = \frac{e_u(\mathbf{p}_t, u(\mathbf{x}_u(\mathbf{p},R_0)))}{R_0}$$

while the corresponding level of utility is the one reached for the Marshallian demand at period 0, equal to $u(\mathbf{x}_u(\mathbf{p_0};R_0))$. We note $\bar{u}_0$ this level of utility.

Consider now that for a reason or another, the need number 1 (first component of the basket vector) is no longer covered by one good but by another one. This old good is replaced by the consumer by a new one in order to cover the same need. Since the two goods are not the same, we cannot assume that, from the consumer point of view, consuming 1 unit of the old good brings the same amount of utility as consuming 1 unit of the new good. Therefore, it is reasonable, in order to model this situation, to consider that for the need number 1, 1 unit of the new good generates an amount of utility equal to α units of the old one. This might be modeled in the following way: let us assume that after the replacement, the utility function from which the consumer takes the decision to consume is v. This function operates on the same set of products covering the same needs, except form the first good which is now the new one. The relationship between u and v is therefore:

$$v(q_1,\mathbf{q}_{(1)}) = u(\alpha q_1,\mathbf{q}_{(1)})$$

where $\mathbf{q}_{(1)}$ stands the vector $\mathbf{q}$ where the first component has been removed. In the previous expression, everything else being equal, if $\alpha > 1$ then the quality of the new good is greater than the quality of the old good, and vice-versa when $\alpha < 1$. While the consumer optimizes from utility function v, its expense function becomes:

[10] We assume that the function is increasing with respect to all its components.
[11] We then assume that the utility function does not change from time to time. But prices change, then quantities as well.

$$\begin{aligned} e_v(\mathbf{p},\bar{v}) &= \min_{\mathbf{q}} \{\mathbf{p}.\mathbf{q}|v(\mathbf{q})=\bar{v}\} \\ &= \min_{\mathbf{q}} \{\mathbf{p}.\mathbf{q}|u(\alpha q_1,\mathbf{q}_{(1)})=\bar{v}\} \end{aligned}$$

The first order conditions of the previous problem can be written[12] (there are n needs covered by the basket ; λ is the Lagrange multiplier):

$$\begin{cases} p_1 &= \lambda\alpha\partial_1 u(\alpha q_1,\mathbf{q}_{(1)}) \\ p_2 &= \lambda\partial_2 u(\alpha q_1,\mathbf{q}_{(1)}) \\ \vdots & \quad \vdots \\ p_n &= \lambda\partial_n u(\alpha q_1,\mathbf{q}_{(1)}) \\ \bar{v} &= u(\alpha q_1,\mathbf{q}_{(1)}) \end{cases}$$

Let us define $\mathbf{r} = (\alpha q_1, \mathbf{q}_{(1)})$. Then, by construction,

$$\begin{cases} p_1 &= \lambda\alpha\partial_1 u(\mathbf{r}) \\ p_2 &= \lambda\partial_2 u(\mathbf{r}) \\ \vdots & \quad \vdots \\ p_n &= \lambda\partial_n u(\mathbf{r}) \\ \bar{v} &= u(\mathbf{r}) \end{cases}$$

$\mathbf{r}$ appears as being equal to $\min_{\mathbf{r}} \{\boldsymbol{\pi}.\mathbf{r}|u(\mathbf{r})=\bar{v}\}$ where $\boldsymbol{\pi}$ is a vector of prices defined by $\boldsymbol{\pi} = \left(\frac{p_1}{\alpha}, \mathbf{p}_{(1)}\right)$. Finally,

$$e_v(\mathbf{p},\bar{v}) = e_u\left(\left(\frac{p_1}{\alpha},\mathbf{p}_{(1)}\right),\bar{v}\right) \tag{15}$$

If we consider a certain level of utility, $\bar{u}$ and if we assume that the perceived quality of the new good is greater than that of the old good ($\alpha > 1$), then we find that if the price vector is unchanged, the expense necessary to reach $\bar{u}$ is lower in the case of the new good than in the case of the old good:

$$\alpha < 1 \Rightarrow e_u\left(\left(\frac{p_1}{\alpha},\mathbf{p}_{(1)}\right),\bar{u}\right) \leqslant e_u\left(\mathbf{p},\bar{u}\right)$$

because e_u is an increasing function with respect to each price component.

The main consequence of equation (15) for our problem is that it is possible to correct prices in order to stay in a constant utility framework even when we allow the goods of the basket to change: even if we don't know the utility function nor the coefficient (α) scaling the quantities in this utility function in case of a quality change, it is possible to correct the vector of price for new goods in order to stay on the same curve of utility. The correction that should be adopted on the price of new goods is exactly the α-coefficient that scales the quantities in the utility function.

For a market in equilibrium, the scale of prices reflects the price differences that make the consumer indifferent to consume a product or another one. If we can assume that the equilibrium is reached at each period of time, if the old good and the new one are sold at the same time, then the price ratio reflects the α coefficient. In other words, the ratio of prices is exactly the value that the consumer makes of the difference in product characteristics.

One may argue about the robustness of the idea of market equilibrium. Nevertheless, from the economic point of view, the Overlapping technique and the Hedonic models both rely on this idea. The only difference between the two is that with respect to this idea, the Hedonic approach should be more robust. Indeed, it should be,

[12] ∂_i stands for the partial derivative with respect to the i^{th} component of the considered function.

by construction, less sensitive to stochastic perturbations that could occur at the level of elementary product prices since Hedonic's relies on a model where stochastic perturbations are taken into account explicitly (the error term of the regression), which is not the case for Overlapping. Besides this, economic theory predicts that, on average, both techniques should lead to same price corrections.

B Econometrics of Hedonic models

The dependent variable in models is the logarithm of the price per unit (the unit is the gram). The calculations are performed over 14 months of observations (from November 2008 to December 2009). A dummy variable for each month is added to the model to take into account price changes during the year. Most of variables characterizing the products are discrete (categorical) ones. The values of some discrete explanatory variables are highly correlated so an additive model is not adequate. For example, the brand of yogurts *Danacol* contains only anti-cholesterol. This defect is corrected by introducing into the model, instead of dummy variables of characteristics, all possible crosses of these terms (interactions). We may introduce in the model all the interactions, even if they are numerous (1 642 for yogurts, 1 149 for chocolate bars), because the number of observations is huge (1.8 millions for yogurts, 1.1 millions for chocolate bars). The structure of models is common to all families of product (i is a product followed in time and t is the time):

$$\log(p_{it}) = c + \sum_{k=1}^{K} \beta_k.\mathbf{1}_{i\in k} + \sum_{m=1}^{13} \gamma_m.\mathbf{1}_{t\in m} + \varepsilon_{it} \quad (16)$$

where β_k is the k^{th} interaction coefficient (there are K possible values), $\mathbf{1}$ is equal to 1 when the condition is true and 0 otherwise. γ_m is the month m fixed effect. 14 months of data are used (1 month is taken as reference).

An interaction corresponds, for example in the yogurt case, to a natural non-organic unsweetened yogurt, with bifidus, full milk and a normal fat content, farm, without additives, which brand is Activia, sold in the chain A and whose amount of material is 500g. With these notations, all possible crossings of characteristics are introduced. p_{it} is the price per gram of the product sold (i) at week t.

For some discrete variables, some values are grouped because there is a very small number of corresponding series. For example, the overall volume for yogurt takes 42 different values (500g, 1000g...). We treat this variable as a discrete variable with only 4 different values (the limits of intervals are 500g, 1000g, 1500 g). Missing values are recoded. For example, the missing values of the variable "Additives" were recoded into "No additives".

B.1 Model for yogurts

The model is computed once for the whole family of products. The variables used in the computation are given in table 12. These 13 variables are then crossed to obtain all possible interactions (1 642). The regressions are computed with these 1 642 interaction terms. The results are given in table 13. The last part of the table shows the estimated values for the coefficient γ_m (month m fixed effect). We see that this coefficient captures essentially the time trend (decrease) of prices: the mean level of prices in November 2009 is about 5.2% lower than in December 2008 (reference month).

B.2 Model for chocolate bars

As for the yogurt model, discrete variables are sometimes reshaped (modalities grouped, missing values coded). For example, the brand variable contains 274 different values. Most of them correspond to sub-brand. For

Table 12: *[yogurts]* Variables used in the Hedonic model

Name of the variable	Number of categories
Store Chain	7
Brand	19
Type of pack	5
Perfume variety	19
Active ingredient	4
Biological information	3
Content of milk	4
Additives	8
Sugar content	3
Process	6
Fat content	2
Type of yogurt	3
Overall volume	5
13 variables	88 categories

example, we grouped all sub-brand Lindt into one Lindt brand. The variables used in the computation are given in table 14.

These 11 variables are then crossed to obtain all possible interactions (1 137). The regressions are computed with these 1 137 interaction terms. The results are given in table 15. The last part of the table shows the estimated values for the coefficient γ_m (month m fixed effect). It seems that there are very small seasonal variations: at the end, the mean level of prices in December 2009 is 0.2% lower than in December 2008. No time trend is observable in this case, by opposition to the yogurt case.

Table 13: *[yogurts]* Hedonic model results

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1 654	202 722,190	122,565	12 416,5	<.0001
Error	1 800 000	17 776,947	0,010		
Corrected Total	1 800 000	220 499,136			

R-square	Coeff Var	Root MSE	Distance Mean	Number of obs
0,919379	-7,441059	0,099354	-1,335207	1 802 558

Source	DF	Type III SS	Mean Square	F Value	Pr > F
crossings	1 641	201 844,2	123,00	12 460,7	<.0001
months	13	449,0	34,54	3 499,2	<.0001

Parameter	Estimated value	Standard-dev	T-test value	Pr > |t|
month 200811	0,006	0,00039	15,89	<.0001
month 200812	0,000	.	.	.
month 200901	-0,003	0,00039	-8,36	<.0001
month 200902	-0,004	0,00039	-10,21	<.0001
month 200903	-0,013	0,00039	-32,54	<.0001
month 200904	-0,001	0,00039	-2,8	0,005
month 200905	-0,015	0,00039	-38,71	<.0001
month 200906	-0,025	0,00039	-64,57	<.0001
month 200907	-0,024	0,00039	-60,63	<.0001
month 200908	-0,023	0,00040	-56,88	<.0001
month 200909	-0,021	0,00040	-54,05	<.0001
month 200910	-0,041	0,00040	-104,02	<.0001
month 200911	-0,052	0,00039	-131,76	<.0001
month 200912	-0,036	0,00040	-91,18	<.0001

Table 14: *[Chocolate bars]* Variables used in the Hedonic model

Name of the variable	Number of categories
Store chain	7
Brand	11
Type of product	6
Type of pack	6
Perfume variety	6
Biological information	3
Fair Trade Information	2
Number of parts	7
Size	5
Additives	12
Overall Volume	5
11 variables	70 categories

Table 15: *[Chocolate bars]* Hedonic model results

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1 149	240 154	209,011	20 905,7	<,0001
Error	1,15E6	11 510	0,010		
Corrected Total	1,15E	251 664			

R-square	Coeff Var	Root MSE	Distance Mean	Number of obs
0,954263	54,97	0,0999	0,18189	1 152 427

Source	DF	Type III SS	Mean Square	F Value	Pr > F
crossings	1136	240 142,6	211,39	21 144,0	<.0001
months	13	11,3	0,87	87,03	<.0001

Parameter	Estimated value	Standard -dev	T-test value	Pr > |t|
month 200811	-0,0011	0,00050	-2,28	0,0224
month 200812	0,000	.	.	.
month 200901	-0,0012	0,00050	-2,31	0,0210
month 200902	0,0023	0,00050	4,58	<,0001
month 200903	-0,0005	0,00050	-1,03	0,3026
month 200904	0,0005	0,00050	1,02	0,3067
month 200905	-0,0074	0,00050	-14,88	<,0001
month 200906	-0,0001	0,00050	-0,22	0,8248
month 200907	0,0031	0,00050	6,08	<.0001
month 200908	0,0029	0,00050	5,82	<.0001
month 200909	0,0020	0,00050	3,97	<.0001
month 200910	-0,0019	0,00050	-3,81	0.0001
month 200911	-0,0073	0,00050	-14,67	<.0001
month 200912	-0,0020	0,00050	-3,94	<.0001

C Some insights into replacements

This annex deals with the concept of equivalent replacement. In the traditional CPI, when the replaced product and the replacing one are closed enough in terms of characteristics, they are considered as comparable and (arbitrarily), no quality correction is made. While a quality correction can be easily computed in all cases of replacement, including the ones where the products could be seen as comparable (i.e. we say in that case that the two products are equivalent), it is interesting to evaluate how far are these last replacements from the equal quality case (the situation where the J coefficient of Eq. 2 is equal to 1 which is what we assume when we say that the two products are directly comparable, or equivalent products). The present annex aims at studying this issue.

Two approaches are possible concerning "Equivalent replacements": the first one consists in systematically applying a quality adjustment, even for type 1-replacements (see section 2); the second one consists in applying a quality adjustment for all the replacements except from type 1 ones (see table 2), assuming, in this last case, that the price of the two products are directly comparable; or, in other words, assuming that the products are equivalent (see section 2).

Table 16 shows the results for yoghurt for the yoghurts CPI's annual increase while type-1 replacements are assumed to be equivalent (for these replacements, the price difference assumed to be a pure price difference and the quality coefficient J is set equal to unity).

Table 16: *[Yoghurts]* Price evolution between December 2008 and December 2009

Type of quality adjustment	Mean annual increase
(1) No quality correction	-4.14% $[-4.5\%\ ,\ -3.8\%]$
(2) Link-to-show-no-price-change (+ equivalent)	-4.01% $[-4.3\%\ ,\ -3.7\%]$
(3) Bridged overlap with real price increase (+ equivalent)	-4.04% $[-4.3\%\ ,\ -3.7\%]$
(4) Last month overlapping (+ equivalent)	-4.07% $[-4.4\%\ ,\ -3.8\%]$
(5) Penultimate month overlapping (+ equivalent)	-4.02% $[-4.3\%\ ,\ -3.7\%]$
(6) Hedonic model (+ equivalent)	-4.03% $[-4.3\%\ ,\ -3.7\%]$

Note: computation made over 100 annual simulations. The intervals in brackets correspond to 95% confidence intervals. Central algorithm of replacement used. No quality correction applied for type 1-replacements.

Comparing table 16 with table 5 allows to observe whether or not, for any quality correction method, the annual decrease of the indices is larger (in absolute value) in this new computation (table 16) than it was in the first (table 5). The largest difference is obtained in the case of Hedonic models where the decrease is -4.03% in the new computation; it was -3.52% in the first computation. Taking into account the distribution of the mean, we cannot reject the equality of the two indices.

Table 17 show the same results for chocolate bars. Comparing tables 9 and 17, we see that the computed indices are consistent statistically, in the sense that we do not reject the null hypothesis in a pairwise equality test. Nevertheless, as in the case of yoghurts, the indices computed with equivalent replacements are a bit higher than that computed with a generalized quality adjustment in case of replacement.

Up to this point, we may suspect that a bias exist for what is called here "equivalent replacements": the indices are not significantly modified by the fact we set the J quality factor equal to 1 for type 1-replacements

Table 17: *[Chocolate bars]* Price evolution between December 2008 and December 2009

Type of quality adjustment	Mean annual increase
(1) No quality correction	1.90% [1.4% , 2.5%]
(2) Link-to-show-no-price-change (+ equivalent)	1.09% [0.7% , 1.6%]
(3) Bridged overlap with real price increase (+ equivalent)	1.08% [0.6% , 1.6%]
(4) Last month overlapping (+ equivalent)	1.08% [0.7% , 1.5%]
(5) Penultimate month overlapping (+ equivalent)	1.14% [0.7% , 1.6%]
(6) Hedonic model (+ equivalent)	1.18% [0.8% , 1.6%]

Note: computation made over 200 annual simulations. The intervals in brackets correspond to 95% confidence intervals. Central algorithm of replacement used. No quality correction applied for type 1-replacements.

but since the frequency of replacement is small (they represent 1% of price transitions taken into account in the computation of the yoghurts price index; 3% in the case of chocolate bars), it does not demonstrate that the type 1-replacements are, on average, free of quality effects. The results for chocolate bars are shown in table 19.

In order to check whether or not a bias exists, we compute the distribution of the J coefficient (see Eq. 2) for all the quality estimation techniques in case of equivalent-type replacement. For yoghurts, the results are shown in table 18.

Table 18: *[Yoghurts]* Mean value of quality factors for all the cases of "equivalent"-type replacements

Type of quality adjustment	Mean
(2) Link-to-show-no-price-change	0.959 [0.942 , 0.979]
(3) Bridged overlap with real price increase	0.961 [0.944 , 0.982]
(4) Last month overlapping	0.966 [0.952 , 0.985]
(5) Penultimate month overlapping	0.960 [0.945 , 0.979]
(6) Hedonic model	0.947 [0.932 , 0.962]

Note: The computed statistics the J correction factor (see Eq. 2) of the considered method. Computation made over the whole cases of type 1-replacement that occur in the 100 annual simulations. The number of cases is 31 161. The intervals in brackets correspond to 95% confidence intervals. Central algorithm of replacement used.

In both cases of yoghurts and chocolate, the 95% confidence intervals of estimated J-coefficient for equivalent replacements do not contain 1 as they should be if the two considered products had the same level of quality. In case of yoghurts, 1 is higher than the upper bound of the interval: the level of quality of the replacing product is overestimated with respect to the replaced product if we assume that the replaced and the replacing products are equivalent. In other words, the quality of what we assume to be here equivalent products is decreasing. At the opposite, in the case of chocolate bars, 1 is lower than the lowest bound of the interval: the level of quality of the replacing product is underestimated with respect to the replaced product if we assume that the replaced and the replacing products are equivalent. In other words, the quality of what we assume here to be equivalent products is increasing.

Table 19: *[Chocolate bars]* Mean value of quality factors for all the cases of "equivalent"-type replacements

Type of quality adjustment	Mean
(2) Link-to-show-no-price-change	1.082 [1.063 , 1.100]
(3) Bridged overlap with real price increase	1.082 [1.063 , 1.100]
(4) Last month overlapping	1.081 [1.062 , 1.098]
(5) Penultimate month overlapping	1.092 [1.074 , 1.110]
(6) Hedonic model	1.076 [1.059 , 1.092]

Note: The computed statistics the J correction factor (see Eq. 2) of the considered method. Computation made over the whole cases of replacement that occur in the 100 annual simulations. The number of cases is 81 223. The intervals in brackets correspond to 95% confidence intervals. Central algorithm of replacement used.

In any case, the notion of equivalent product used here as a rather arbitrary property seems to be poorly supported by facts.

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