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SEQUENTIAL STOCHASTIC DOMINANCE, PRINCIPLES AND IMPLEMENTATION. AN APPLICATION TO THE ASSESSMENT OF THE FRENCH AND GERMAN TAX SYSTEMS

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Abstract

This paper presents the principles of stochastic dominance which is a powerful tool that enables to have a generalized approach for the measure of poverty and inequalities.

This paper first summarizes the literature on stochastic dominance with all its latest developments. We then present how to apply stochastic dominance to the assessment of the redistribution of a tax system and discuss of the constraints that are bound to it. A tax system is designed to address equity concerns, but two dimensions have to be taken into account. The first one is the redistribution between high income and low income (vertical redistribution) and the second one is the redistribution from small families towards large families (horizontal redistribution). Therefore, we study and discuss the uses of sequential stochastic dominance (Atkinson and Bourguignon (1989)) and restricted dominance (Chambaz and Maurin (1998)) that seem well suited to the assessment of a tax system.

We finally provide SAS macros with the computations of all the stochastic dominances that are contained in the paper. We also provide an example of how to use the macros to compare the tax systems between France and Germany.

Key Words: Stochastic dominance, redistribution, tax system, inequality.

Résumé

Ce document de travail présente les principes et l'implémentation de la dominance stochastique, un puissant outil permettant une approche générale de la mesure des inégalités et de la pauvreté.

Ce papier résume dans une première partie la littérature sur la dominance stochastique et ses développements les plus récents. Nous présentons ensuite comment appliquer la dominance stochastique pour évaluer le caractère redistributif d'un système fiscal et discutons des contraintes que cela pose. Il est en effet important de distinguer deux dimensions dans un système fiscal. La première dimension concerne la redistribution verticale, c'est-à-dire la redistribution des ménages à hauts revenus en faveur des ménages à bas revenus. La dimensions horizontale concerne la redistribution en faveur des ménages de grande taille. Dans cette perspective, nous étudions et discutons de l'utilisation de la dominance stochastique séquentielle (introduite par Atkinson et Bourguignon (1989)) et de la dominance stochastique restreinte (voir Chambaz et Maurin (1998)) qui semblent être particulièrement bien adaptées à l'évaluation d'un système fiscal.

Finalement, nous mettons à disposition des macros SAS qui permettent de calculer l'ensemble des dominances stochastiques présentées dans ce document de travail. Nous proposons également un exemple d'utilisation dans le cadre d'une comparaison entre la France et l'Allemagne.

Mots clés : dominance stochastique, redistribution, taxation, inégalités

Introduction

Income is most of the time known at the household level. Measuring income inequalities ignoring differences among households along other dimensions than mean income (such as household composition) leads to highly distorted pictures, especially for inter-temporal or cross-country comparisons.

The use of equivalence scales enables one to introduce household composition in the study of income inequality. Indeed, income per adult-equivalent, an improved version of per capita income, can take accurately into account the economies of scale realized when individuals live together. For instance the French and German National Statistical Offices as well as Eurostat use the `modified OECD equivalence scale' which leads to divide the income of a couple with two children by $1 + 0,5 + 2 \times 0,3 = 2,1$ rather than by 4. This approach entails a number of problems which one may want to set aside. The theoretical grounds on which equivalence scales rely are still disputed (see Lechêne (1993)) and the empirical determination of the scale itself is far from being straightforward and often relies on additional hypotheses (See for instance Hourriez and Olier (1997) for an estimation in the case of France). The use of a uniform equivalence scale (with weights constant throughout the distribution) is for instance obviously controversial as it relies on normative assumptions, which are most of the time note openly revealed. But most of all, the parsimony principle is violated when one uses equivalence scales to compare income distributions on populations of heterogeneous households: comparisons can be made with less restrictive hypotheses. Sequential stochastic dominance provides the framework for such comparisons. It relies on a bivariate approach: utility is not a function of the sole (possibly per adult equivalized) income, but it is a function of both income (increasing) and needs of the household (decreasing). The needs of the household are, most of the time, proxied by its composition[2].

Another justification of the stochastic dominance approach is the following: as soon as studying income distribution in the end leads to public policy choices (like proposals for tax-benefit reforms), one would like the choice to be agreed upon by a wide range of individuals with possibly differing preferences with regard to redistribution. These differing preferences can be summarized within differing social welfare functions (swf). And the reforms can then be evaluated if they lead to a better (distribution) outcome for a wide range of swfs. The swfs can just be related to income (univariate case / overall dominance) or be related to both income and needs (bivariate case sequential dominance).

Nevertheless equivalence scales rely on a lot of assumptions and one may want to relax these assumptions in order to take into account household composition in the most general possible way. This is what stochastic dominance enables one to do.

1 Stochastic dominance: a survey of the basic result

[2] Note that other factors of need, which are of interest for social choice theory like invalidity for instance, can be taken into account following this approach

In this section, we first present the basic (univariate) setting for (overall) stochastic dominance. We then introduce heterogeneity among households and define sequential stochastic dominance. Lastly we present some relevant extensions.

1.1 Overall dominance

Let y be the continuous income variable over a bounded range $[0, y_{\max}]$, with probability density function (henceforth *pdf*) f and cumulative distribution function (henceforth *cdf*) F. Following Atkinson (1970), comparisons of inequality levels between two distributions can rely on social welfare functions (swf)[3]. In the simple case of symmetric and additively separable functions, they can be written:

$$\mathbb{W} = \int_0^{y_{max}} u(y)f(y)dy$$

Overall First Order Stochastic Dominance (OFOD) states that a distribution is preferred (meaning that it leads to a greater social welfare) to another if and only if the cdf of the preferred distribution is never above and at least once strictly beneath the other distribution. In order for this criterion to hold, the utility function must be assumed non-decreasing $u'_y(y) \geq 0 \ \ \forall y$ **(P1)**. This can be written:

$$\boxed{[\text{OFOD :}] \text{ if } \mathbf{P1} \text{ then: } f \succ_{\mathbb{W}} f^* \Leftrightarrow \forall y \ F(y) \leq F^*(y)}$$

Note that this approach can be extended to absolute monetary poverty: if the poverty line is defined exogenously, then the OFOD criterion is equivalent to saying that the poverty rate for the preferred distribution is lower, whatever the poverty line is.

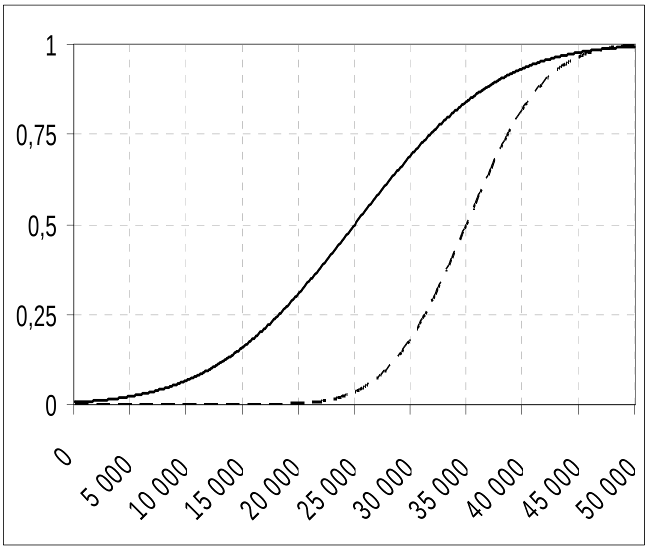
Unfortunately, in a wide number of cases --- as soon as the cdf cross each other --- the OFOD cannot conclude. Assuming slightly more restrictive characteristics of the utility function, a more powerful criterion can be derived, namely Overall Second Order Stochastic Dominance (OSOD). According to that criterion, a distribution is preferred to another if and only if the surface under the cdf of the preferred distribution is never greater and at least once strictly smaller than the surface under the other distribution. This criterion entails the additional assumption that, on the top of the utility function being non-decreasing, its first derivative has to be non-increasing $u''_y(y) \leq 0 \ \ \forall y$ **(P2)**. Intuitively, this criterion can deal with some crossing cdf. OSOD can be written as follows:

$$\boxed{[\text{OSOD :}] \text{ if } \mathbf{P1,P2} \text{ then: } f \succ_{\mathbb{W}} f^* \Leftrightarrow \forall y \ \int_0^y F(t)dt \leq \int_0^y F^*(t)dt}$$

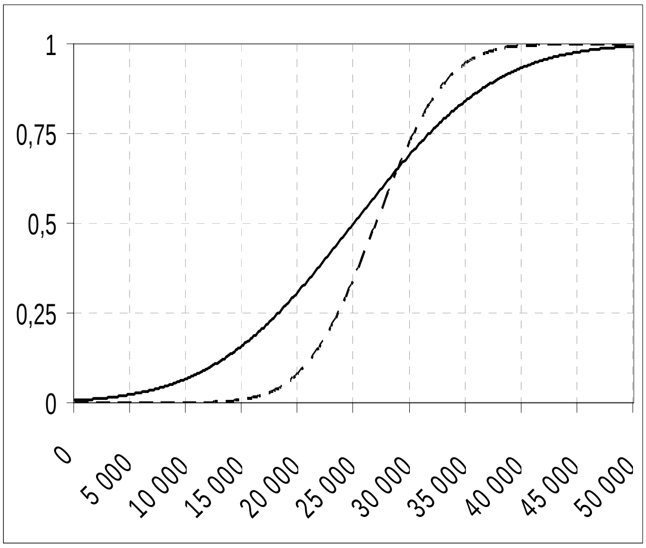
[3] Of course, a different approach leads to `summarize' the distribution with a real number (an `inequality index', which has to respect some normative properties) but the cost is a huge loss of information. The approaches presented here all lead to partial orders but with pretty consensual assumptions over the comparison criterion. In a classical trade-off setting, extending the subset of comparable distributions entails making ever more restrictive (ever less consensual) assumptions over the criterion.

Coming back to a poverty setting, OSOD means that the poverty gap of the preferred distribution is lower, whatever the poverty line is.

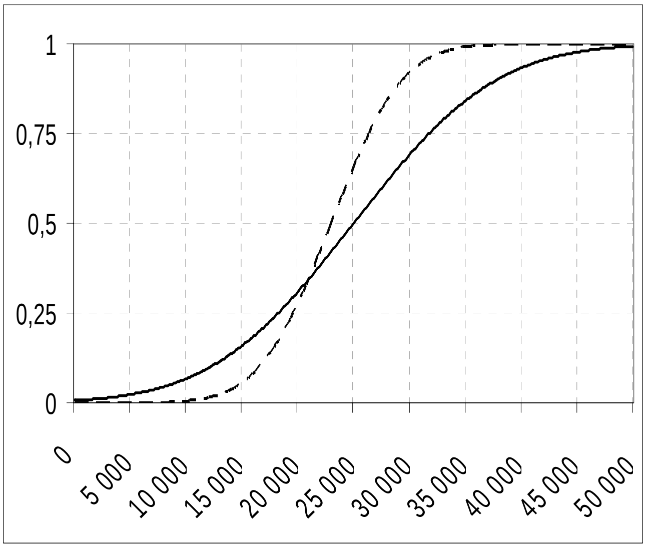
Foster and Shorrocks (1988) showed that OFOD was a sufficient but not necessary condition for higher order dominances. Higher order stochastic dominance criteria can be defined, and it can be shown (Davidson and Duclos (2000)), that *s* order stochastic dominance is equivalent



(a) FO



(b) No FO but SO



(c) Neither FO nor SO

Figure (1) Stochastic Dominance

to comparing Foster-Greer-Thorbecke P_{s-1} for all possible poverty lines. Figures 1(a) to 1(c) show the various situations possible.

[Figure 1abc]

1.2 Dominances and the Lorenz curve

The seminal presentation of Atkinson as been extended since Shorrocks (1983) introduced the equivalent concept of Generalized Lorenz Dominance (GLD) which consists in comparing the Lorenz curves, re-scaled by the mean income of the distribution. This concept is more intuitive than OD and explicitly addresses the trade-off between equity and efficiency underlying income distribution comparisons.

First, pure equity concerns are captured by requiring mean-preserving regressive transfers not to increase welfare. Following Dasgupta, Sen and Starrett (1973), this is equivalent to assuming the swfs to be Schur-concave[4] **(L1)**. In this setting, two distributions with equal means can be compared if their Lorenz curves do not intersect. The distribution corresponding to the Lorenz curve lying above is preferred. If the Lorenz curves intersect, no conclusion can be reached. Formally let L(**y**,p) be the Lorenz curve of the distribution **y**, with mean μ; $p \in [0,1]$. We then have:

$$\boxed{[\textbf{LD} :] \text{ if } \mu = \mu' \text{ and } \textbf{L1} \text{ then: } W(\mathrm{y}) \geq W(\mathrm{y}') \Leftrightarrow L(\mathrm{y}',p) \geq L(\mathrm{y},p)\ \forall p}$$

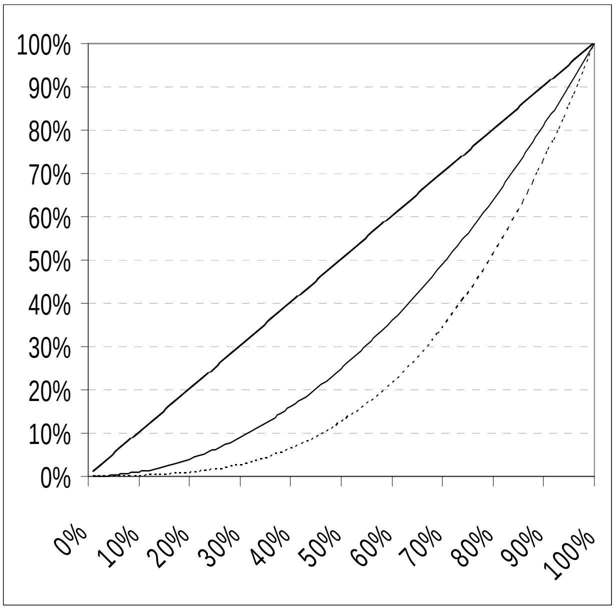
In order to introduce the efficiency dimension, we need to generalize this setting to the case where means differ. Indeed, higher mean income (efficiency) may possibly offset the impact of higher inequality (inequity) in the comparison. To take this aspect into account we have to further assume that the swf be increasing **(L2)**[5]. In this setting, two distributions with differing means can be compared if their generalized Lorenz curves do not intersect. The generalized Lorenz curve is the Lorenz curve re-scaled by the mean of the distribution. The distribution corresponding to the generalized Lorenz curve lying above is preferred. If the generalized Lorenz curves intersect, no conclusion can be reached. Let GL(**y**,p) be the generalized Lorenz curve of the distribution **y**, with mean μ; $p \in [0,1]$. We then have:

$$\boxed{[\textbf{GLD} :] \text{ if } \textbf{L1,L2} \text{ then: } W(\mathrm{y}) \geq W(\mathrm{y}') \Leftrightarrow GL(\mathrm{y}',p) \geq GL(\mathrm{y},p)\ \forall p}$$

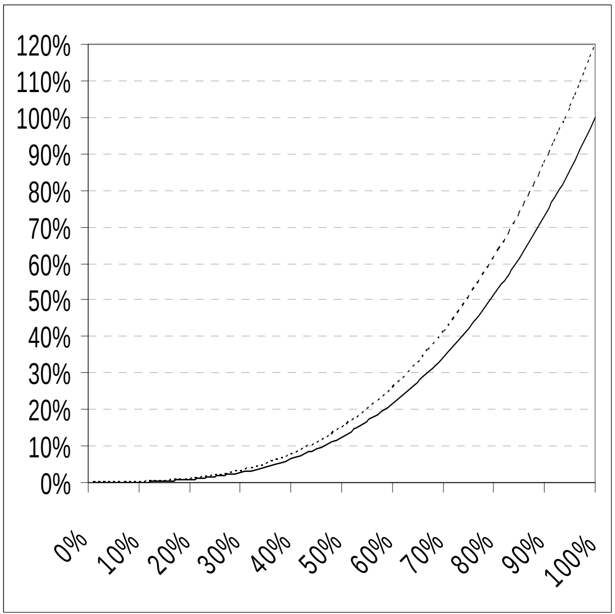
A sufficient condition for having GLD is when a distribution has both higher mean and higher (simple) Lorenz curve. Figure 2(a) and 2(b) illustrates how rescaling a more unequal distribution which has higher mean can lead it to GL-dominate although it was L-dominated. Figure 2(c) shows that even the most equal distribution is in the end GL-dominated by a distribution with slightly higher mean.

[4] Schur-concavity is a weak form of concavity. A function is Schur-concave if it translates the ordering of vectors according to majorization into the standard scalar ordering.
[5] Note that **L1, L2** are verified by utilitarian swf complying with **P1, P2**.

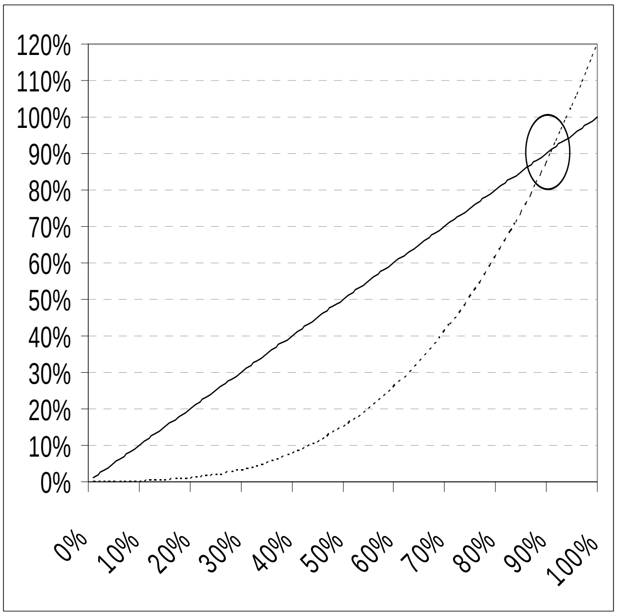
[Figure 2abc]



(b) LD



(b) GLD (1)



(c) GLD (2)

Figure (2) Lorenz Dominance

1.3 Dual approaches

The approaches in terms of Generalized Lorenz Dominance and in terms of Second Order Stochastic Dominance are dual. Following Atkinson and Bourguignon (1989), we present in this section this dualism with a common formalism.

1.3.1 "Primal approach" and the OSOD

The approach in which swf is defined in terms of ``expected utility"

$$W = \int_0^{y_{max}} u(y)f(y)dy$$

with $u' \geq 0$ and $u'' \leq 0$ leads to the following criterion

$$\Delta \int F(y)dy = \Delta \int_0^{y_{max}} \left(\int_0^y f(x)dx \right) dy \geq 0$$

1.1.1. "Dual approach" and the GLD

The criterion relying on GLD

$$\Delta \int_0^1 F^{-1}(p)dp = \Delta \int_0^{y_{max}} F^{-1}(F(y))dF(y) = \int_0^{y_{max}} yf(y)dy \geq 0$$

corresponds to an approach where swf is defined with respect to ranking:

$$W = \int_0^1 w(p)y(p)dp$$

where $w'' \leq 0$. It means that the weight w attached to the level y of income attained by individuals with relative rank p in the population is decreasing with the rank (and with the level of income since $p = F(y)$ with F increasing. Atkinson (1970) was the first to make the link between the two approaches.

1.3.2 Some trivial results to keep in mind

1. OFOD implies OSOD and therefore GLD

2. Comparing distributions with the same means, LD and GLD and, therefore OSOD are equivalent

3. When a distribution has a higher mean income, it cannot be GL-dominated by another distribution with lower mean income: more income can compensate more inequality, but the reverse does not hold (compare Figures 2(b) and 2(c)). Since GLD and OSOD are equivalent, and OSOD is necessary for OFOD to hold, when a distribution has a

higher man income, it can be neither OFO- not OSO-dominated by another distribution.

This latest simple result is of great importance for the practical implementation of stochastic dominance (see section 3).

1.4 Political economy concerns: robustness, consensus and "restricted dominance"

One of the advantages of the approach presented above is the low degree of specification of the swf underlying the criterion. As we have seen only S-concavity is requested, which is pretty general. What are the implications of this generality gain? We focus here on the comparison of distribution before and after redistribution (for the difficult assessment of inter-temporal or cross-country inequality variation, see section 3.2).

1.4.1 Ex-ante design of redistributive policies

Trying to design a tax-transfer system, and ignoring the real swf it is facing, the government may want to implement a reform implementing a GL-dominating distribution (either over the pre-tax-and-transfers distribution, or, as in Atkinson and Bourguignon (1989), over the actual post-tax-and-transfers distribution). Thus GLD helps to design reforms which are robust to our uncertainty over the real social preferences we are facing. Clearly, since GLD is a partial ordering only, the set of possible reforms is reduced by the desire of implementing a robust reform. This is the price to pay for robustness.

1.4.2 Ex-post evaluation

Evaluating ex-post either a reform, or a tax-transfer system, the economist may want to formulate judgements being as general as possible with regard to the swf underlying the analysis. The price to pay here is the eventuality for the analyst not to be able judge the reform or the system.

1.5 Consensus

Now clearly the generality over the swf models the fact that we ignore the real swf, not that there may be heterogeneity within the population. In promoting a GL-dominating reform, one can be sure that the next government with differing views (trying to implement a reform enhancing a welfare measured according to another swf) will still agree on his or her reform being welfare enhancing[6].

Nevertheless, if people can vote on my reform and have different views on the inequalities, the present framework does not help to conclude whether or not they will support the reform.

[6] We do not here engage in trying to address the tricky issue of cardinal measurement of the welfare gain and restrict ourselves to ordinal concerns.

To keep things simple we remain in a utilitarian %framework with symmetric, additively separable swf.

One way of constraining the criterion to gain comparability consists in writing the integrals of the criteria up to a given level say $\overline{y}$ (or in the dual approach up to a given percentile $\overline{y} = F(\overline{y})$) and to speak of dominance "up to $\overline{y}$". This can be called **``restricted dominance''** (see Atkinson and Bourguignon (1989)). If a reform or a tax-transfer system leads to dominating distribution ``up to the last decile'' for instance, the top decile of the population above the threshold is simply ignored in the evaluation. Ignoring in first approximation possible re-ranking across the threshold between the two distributions, one can thus state that the reform planned is ``supported'' by any well behaved swf of the bottom 90% of the population. The top decile may have different views, this will not be taken into account. This approach thus for instance allows for some heterogeneity of the individual utilities within an additive swf.

Conversely, a system which only leads to a dominating distribution ``up to the fourth decile'' is clearly not robust in case a democratic vote is cast upon it. One cannot exclude that the effective swf in the population leads more than 50 % individuals to vote for it, but there is at least one swf (and it may be the ``true one'') for which a 60 % block of the population may vote against the project.

Note that using restricted dominance criteria also loosen the necessary condition imposed on the mean income of the dominating distribution: only the mean income of the bottom x\% of the dominating distribution has now to be higher. William Gates may be taxed heavily and the tax revenue burnt, it will not affect the dominance ``up to William Gate'' (assuming Mr. Gate remain the richest individual, since our swfs obviously remain anonymous).

1.6 Sequential Dominance

We now come to income distributions on populations of heterogeneous households. We introduce a `needs' variable which is independent of the income variable. Atkinson and Bourguignon (1982} study the most general bi-continuous case. Atkinson and Bourguignon (1987) restrict themselves to discrete needs, which do not need to be cardinal. Bourguignon (1989) and Atkinson and Bourguignon (1989) implement this latter approach in the case where need is the size of the household. We quickly present both discrete approaches (qualitative and quantitative) approaches here.

Let the population be divided into n subsets generated by the `need' variable. Let p_i be the share of the i-th subset in the total population $\sum_{i=1}^{n} p_i = 1$. We suppose that the index i ranks the subsets by increasing need level (or equivalently by decreasing well being level, conditional on income, i can thus be thought of as the size of the family, for the sake of the intuition). Group n is the `most deserving group'. $u^i(y)$ denotes utility function of households of the subset i. Remaining within a setting of symmetric and additively separable swfs, we have:

$$\mathbb{W} = \int_0^{y_{max}} u(y)f(y)dy = \sum_{i=1}^{n} p_i \int_0^{y_{max}} u_i(y)f_i(y)dy$$

with $f_i(y) = f(i/y)$.

Now we need to make assumptions on how u^i varies with i. If no hypothesis is made (which means that we assume that no agreement *at all* can be reached within the population on the way u^i varies with i), then the only way to compare the different groups is to require dominance for each group taken separately. As Atkinson and Bourguignon (1989) put it "this is highly restrictive and precludes any redistribution between family types"[7].

Let us suppose now that it is possible to rank marginal (income) utilities according to the reverse ranking according to which (income) utilities are ranked. Intuitively, if the `need' variable is the size of the household, then it means that we assume that marginal (income) utilities are increasing with the size of the household. For instance, conditional on income, large families have a lower utility but a higher marginal utility than singles. This is quite natural if we assume that marginal utilities are non-increasing.

In the qualitative framework, this assumption (**P3**) can be written as follows:

$$\forall y\ \forall i = 1, ...n,\ u_y^i(y) = \sum_{k=1}^{i} \varepsilon^k(y) \text{ où } \varepsilon^k(y) \geq 0\ \ \forall k\ \forall y$$

In the quantitative framework, we have simply:

$$\forall y\ \ u_y^i(y) \text{ non-decreasing with } i$$

First Order Sequential Dominance can then be written as:

$$\boxed{[\textbf{SFOD :}] \text{ if } \textbf{P1,P3} \text{ then: } f \succ_{\mathbb{W}} f^* \Leftrightarrow \left(\forall y\ \forall j = 1, ..., n\ \sum_{i=j}^{n} p_i F(y) \leq \sum_{i=j}^{n} p_i F^*(y)\right)}$$

Practically it means that we successively verify the FOD on the increasing suite of subsets, beginning with the population with the higher needs (and which thus has, conditional on income, the higher social marginal utility) and adding step by step to this core population the sub-populations with lesser needs. The last FOD to be verified is obviously the OFOD. Therefore SFOD strictly implies the OFOD.

Like for the overall dominance criteria, some additional restrictive hypotheses on utility functions (i.e. limiting the range of the possible consensus on a `dominant' reform) lead to a more general dominance criterion (closer to a complete order). Let us suppose that the differences between two successive marginal (income) utilities are decreasing with `needs'.

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[7] At the other extreme, using equivalence scales means that the way u^i varies with i is completely determined (or subject to consensus in the population), and that we can replace $u^i(y)$ by $u(y/e_i)$ where e_i is the scale.

In the qualitative framework, this assumption (P4) can be written as:

$$\forall y, \forall k = 1, ..., n \ \varepsilon_y^k(y) \leq 0$$

In the quantitative framework, we have simply:

$$\forall y \ \ u_{yy}^i(y) \text{ non-increasing with } i$$

Second Order Sequential Stochastic Dominance can then be written as:

$$\boxed{[\text{SSOD :}] \text{ if } \mathbf{P1,P2,P3,P4} \text{ then: } f \succ_W f^* \Leftrightarrow \left(\forall j = 1, ..., n \ \forall y \ \sum_{i=j}^{n} p_i \int_0^y F(t)dt \leq \sum_{i=j}^{n} p_i \int_0^y F^*(t)dt \right)}$$

We now focus on `needs' being defined as the size of the family. If we have **P1-P4** then we can write **P5**[8]:

$$u_y^{i+1}\left(\frac{i+1}{i}y\right) \leq u_y^i(y)$$

this means that `the marginal utility of income decreases with family size at constant income per capita'. We can now write another version of the SOSD:

$$[\text{SSOD}^* \text{ :}] \ f \succ_W f^* \Leftrightarrow \ \begin{array}{l} \sum_{i=1}^{n} p_i \int_0^{y_i} F(t)dt \leq \sum_{i=1}^{n} p_i \int_0^{y_i} F^*(t)dt \\ \forall y_i \text{ verifying } \frac{i}{i+1} y_{i+1} \leq y_i \leq y_{i+1} \end{array}$$

This, as shown in Fleurbaey et *al.* (2003) can lead back to an approach in terms of generalized equivalence scales.

1.7 Necessary condition for SSD to be verified

Since a higher mean income of the dominating distribution is a necessary condition for the criterion to be verified at each step of the sequential process, the average redistribution between household groups is constrained. As soon as the mean income of one of the sets decreases, the criterion fails. This implies first of all the neediest group cannot see its mean income decrease. Then, the adjunction of the second neediest group cannot lead the mean income of the reunion of the two groups to decrease. This means that, although the mean income of the second-worse off group may decrease, it should be at least offset by a rise of the mean income of the neediest group, in order for the average of the two to remain at least constant. And so on. The average inter-group redistribution thus has to take place top-down, from the less needy to the needier.

[8] See Bourguignon (1989), p.70.

1.8 Restricted SSD and the question of the cut-off point

The sequential dominance approach can also be restricted to gain comparability. The definition of a constant cut-off line across the household types is nevertheless not an easy one (see section 3 for more details).

1.9 Extension: Equivalence scales

P1 and **P2** are quite consensual hypotheses, but **P3** and **P4** are more likely to be controversial since they assume comparability of marginal utility levels across households.
One solution to relax these hypotheses is to make use of `fuzzy equivalence scales'. The basic idea is that a consensual comparison of two different households will always be reached if incomes are sufficiently different. For instance, compared to a single with \$1 000, a couple with \$1 000 will always be considered worse off and a couple with \$2 000 better off. Between these bounds however, no consensus might be reached. Nonetheless, the bounds constitute a structure which can be used to compare income distributions on heterogeneous populations: some intermediate point between `classical' (non-robust) equivalence scale and no equivalence scales at all.

Taking as a reference category \$i=1\$, the considerations above lead to the following formal setting:

$$\left.\begin{array}{l}(\mathbf{P3'a}):\ u_i'(\alpha_i y) \leq u_{i+1}'(y)\ \forall y \in \mathbb{R}_+,\ \forall i \in 2,...,n \\ (\mathbf{P3'b}):\ u_i'(\beta_i y) \leq u_{i+1}'(y)\ \forall y \in \mathbb{R}_+,\ \forall i \in 2,...,n\end{array}\right|$$

Note that **P3** is a particular case of **P3'a**, with $\alpha_i = 1$. Note that **P5** is a particular case of **P3'b** with $\beta_i = \frac{i}{i-1}$. Thus the set of utility functions considered in Bourguignon (1989) (defined by **P1-P5** which are implied by **P1-P3'a&b**) is a superset of the utility functions considered here, which are thus less general than those considered for the SSD. Fleurbaey *et al.* (2003) show that under these assumptions a generalized SSOD $\succ_{\alpha,\beta}$ can be written, which depends on the vectors $(\alpha)_i$ and $(\beta)_i$, in R^n.

The classical equivalence scale approach implies the definitions of a vector $e = (e_1,..., e_2)$ of deflators (often $e_1 = 1$). In this framework we can define the swf:

$$\mathbb{W} = \sum_{i=1}^{n} p_i e_i \int_0^{y_{max}} f_i(y) u\left(\frac{y}{e_i}\right) dy$$

with *u* verifying **P1** and **P2**. Obviously, another dominance can be defined in this setting, depending on *e*.

Let us now write:

$$\Theta(\alpha, \beta) = \{(e_1, ..., e_n) | \forall i = 2, ..., n,\ \alpha_i e_{i-1} \leq e_i \leq \beta_i e_{i-1}\}$$

it is then possible to define a dominance ($\succ_{\Theta(\alpha,\beta)}$) (namely over the whole set of e within $\Theta(\alpha,\beta)$). This amount to define an overall dominance uniformly over a set of equivalence scales --- the set depending on (α,β). Fleurbaey *et al.* (2003) show that:

$$\boxed{f \succ_{\Theta(\alpha,\beta)} f^* \Leftrightarrow f \succ_{\alpha,\beta} f^*}$$

Thus the two approaches (sequential dominance vs. equivalence scales) meet at this point where the Bourguignon (1989) setting is restricted (and the order is thus less partial) and the equivalence scale paradigm is weakened (the complete order is lost but some consensus over the ranking is obtained).

2 Advantages and pitfalls of using stochastic dominance

2.1 Measuring inequalities

2.1.1 Inequalities and equivalence scales

The measure of income inequalities is a two-dimension problem. The first one concerns households of different sizes and compositions and the second one regards pure income inequalities. It is now common practice to use equivalence scales to reduce the problem to one dimension thus making different households incomes comparable. The underlying rationale of equivalence scale is to take into account the economies of scales within large households. However there exist many different ways of choosing an equivalence scale. First, one can use a normative scale which is devised by experts such as the “modified OECD equivalence scale” now used by many countries. The second type of scales are the ones implicit in the social security system, either in the tax schedule (for instance the French family-splitting) or in the benefit transfers. The third kind of equivalence scales are the ones estimated from the household budget surveys which reveal the consumption patterns of households. Finally, the last method found in the literature is based on subjective welfare measurement. There exists therefore a wide range of methods to compute equivalence scale which lead to a wide range of results.

As was quoted by Atkinson and Bourguignon (1989}, a survey by Whiteford in 1985 tabulates 44 different estimates of a scale for a single person, ranging from 49 to 94 percent of the couple scale. For a couple with two children, he quotes 59 estimates varying from 111 percent of the scale for a couple to 193 percent. The geometric means of these estimates is 138 percent, but the OECD, for example, uses a figure of 159 percent. Regarding France, (Hourriez and Olier (1997)) assessed equivalence scales with different methods. Their concluding remarks spoke in favour of a revision of the standard Oxford scale used in those times. They argued that the Oxford scale was no longer suitable because it underestimated the economies of scale in the contemporaneous societies. The consumption patterns have evolved over the last decades, especially regarding the share of food in the household budget. Thus lower coefficients for each extra household member (i.e. higher economies of scale) would better fit reality. More recently, Koulovatianos et *al* (2005) showed that equivalence scales should be income dependent because the consumption economies of scale change as living standards go up. The intuition underlying this feature is that the income share dedicated to an extra member in the household decreases with the income level. Therefore, no consensus exists on the use of one particular equivalence scale.

2.1.2 from equivalence scales to stochastic dominance

When it comes to deal with inequalities, most papers use aggregate indicators such as the Gini, Atkinson or Theil indicators and each of them need equivalence scales to make every income comparable. Those indicators have two major drawbacks. First of all, they are unitless (i.e. independent of the level of the measured quantity). They solely measure the inequality of a given distribution. They are normative indicators and the lower the indicator, the lower the inequalities. For instance, according to this kind of indicators, Slovakia has fewer inequalities than France. However, in this case, the levels of the two distributions matter. This is directly related to the trade-off between equity and efficiency (see section 2.3). Those indicators do not take efficiency into account, they only focus on equity. On the contrary, SD is not independent of the level of the measured quantity. If two distributions have different means, OSOD analyses the trade-off between efficiency and equity in terms of welfare enhancement (provided that SD gives an ordering), the second drawback concerns the use of equivalence scales. The weight attached to the welfare of children is likely to be a matter of social judgment and they are likely to differ. It is therefore preferable to adopt an approach that treats differences in social judgments in a parallel manner to the way that Lorenz dominance treats different distributional judgments in the income dimension. The use of sequential stochastic dominance (SSD) makes explicit the redistribution between different family types which may be concealed by the use of equivalence scales.

As a conclusion SD provides a more general framework than equivalence scales. Whereas standard indicators focus on inequalities, SD assesses welfare enhancement. In the next section, we will study the advantages and drawbacks of equivalence scales versus SD for inter-country and inter-temporal comparisons.

2.2 Inter-temporal and inter-country comparisons

2.2.1 Inter-temporal comparisons

Many studies focus on the evolution of inequalities over time, trying to determine whether they increase or decrease, in the long run or in the short run. The use of equivalence scales supposes that the underlying assumptions regarding their estimation method have not significantly changed. If it uses consumption patterns then they should be the same. If it is based on subjective welfare, the preferences should be the same. Over the short run, these assumptions seem reasonable, however, over the long run, they are more questionable as Hourriez and Olier (1998) pointed it out (see section 3.1.1).

SD assumes that the population can be divided in different categories that can be unambiguously ranked according to their needs. Thus dominance results are restricted to cases where the marginal distribution of needs is fixed in the distributions being compared.

Moreover, when dealing with inter-temporal comparisons, one has to take inflation into account. Obviously, when comparing two distributions across time, the past distribution needs to be actualized by the rate of inflation in order to have two comparable distributions in terms of real income. In France for instance, the real income mean is increasing in average. A According to the SD properties (see section 2.3), an increase in inequalities can be offset by an increase in the real income, leading to a welfare enhancement, whereas the converse does not hold. Therefore, any ordering based on the dominance criterion will be dependent of inflation. On the contrary, standard indicators using equivalence scales solely focus on inequalities regardless of the real income evolution.

2.2.2 Inter-country comparisons

The second question related to inequalities concerns inter-country comparisons. The choice of an official equivalence scale for inter-country comparisons seems very controversial. Each national equivalence scale has its own country specificity. Nonetheless a study by (De Vos and Zaida (1997)) showed that the ranking of countries of the European community with respect to overall poverty is hardly affected by the use of different equivalent scales. However, and that seems to be a strong pattern of equivalence scales, it leads to huge differences in terms of poverty composition. Nevertheless, these results concern European countries which are rather homogenous. If we extend such comparisons to other countries, the choice of equivalence scale seems to affect poverty, inequality and therefore the ranking of countries according to those indicators (Buhmann et *al* (1988)).

How does SD perform in inter-country comparisons? Using SD to determine which distribution dominates the other one faces too many restrictions. First, inter-countries comparisons raise the problems of purchasing power parity. The exchange rate between the two countries can fluctuate and therefore lead to controversial results from one year to the next. Tackling this problem is possible, by restraining this kind of study to European countries for instance (Chambaz and Maurin (1998)). Similarly to inter-temporal comparisons, inter-country comparisons raise the problem of needs ordering as well as the means of the income distributions. Once more if one of the distributions has a higher mean then, if a ranking is possible, it will probably dominate the other one even if it is more unequal.

Finally, one of the most important problems raised by inter-country comparisons concerns income definition. It is actually difficult to have a homogenous income definition among different countries which is essential for a comparison to be consistent.

The restrictions facing SD make it unsuited to inter-temporal and inter-country comparisons. Standard indicators with equivalence scales perform better, unfortunately the results are not robust to a change in the scale choice.

2.3 Assessing the impact of tax systems

2.3.1 Stochastic dominance and redistributions

The tax system is designed to address equity concerns, but two dimensions have to be taken into account. The first one concerns the redistribution between high income and low income (i.e. vertical redistribution), and the second one the redistribution from small families towards large families (i.e. horizontal redistribution). As was pointed out in the previous section, the use of equivalence scales precludes any assessment of horizontal redistribution. On the contrary, all the restrictions concerning stochastic dominance make it more suited to assessing the redistributive effects of taxes and benefits at a given time period. In particular, sequential dominance enables one to assess explicitly the horizontal component of redistribution whereas OFOD and OSOD address solely vertical redistribution. Usually, the need ordering is given by household size even if the case of single-parent families needs some care. In the next section, we give some advices on the good practices that anyone should keep in mind while dealing with stochastic dominance and redistribution.

2.3.2 Good practices

Even if stochastic dominance seems to be well suited to assess the redistributive effect of the tax system, there are some constraints to care about. First, a constant pattern of tax systems is a decrease between the means of the pre-tax and post-tax distributions. As a matter of fact apart of its redistributive function, the tax system is also meant to produce tax revenue for government expenditures. Therefore, as was pointed out in the second part, even if the second distribution is the most equal possible (i.e. if everyone has the same post tax income), it will never dominate the pre-tax distribution. As we will see in the next section, that is what we obtain whether in France or Germany. More generally OFOD and OSOD will never be verified up to the top of the distribution. OFOD will fail because there is inevitably an income level above which the post-tax income is below the pre-tax income and therefore the two cdf cross. OSOD will also fail because the mean of the post-tax distribution will be below the mean of the pre-tax distribution. Therefore, it leaves us with two options, either to find a device to make the redistribution mean preserving, or to use restricted dominance (see section 2.5).

Our first option is to create a mean-preserving redistribution. The first idea would be to re-scale the post-tax distribution to its pre-tax level. This is equivalent to Lorenz dominance. However, in such a case, the post-tax distribution dominates the pre-tax distribution, which is a logical result as long as the tax schedule is progressive. This result simply states that the post-tax distribution is less unequal than the pre-tax distribution. Therefore this device does not help to compare the redistributive properties of two tax schedules.

A second idea would be to compensate the post-tax income by adding a lump sum transfer to each household in order to have the same mean than the pre-tax distribution. The economic legitimacy of such a transfer would be that the tax revenue enables the government to enhance public services that benefit equally to he whole population. Obviously, the major drawback of this option is that the population does not, actually, equally share the benefits. Apart from this criticism, this assumption seems too strong because the post-tax distribution will mechanically dominate the post tax distribution, as long as the tax system is progressive (because a translation does not modify inequality).

A third idea would be to constrain the redistribution to be mean preserving for each need category. However this hinders any horizontal redistribution which is not realistic. A more realistic device would be to constrain redistribution to lead to non-decreasing means for the post-tax distributions for each embedded sequence of subset of the population. More precisely, we constrain redistribution for the more needy group to be means non-decreasing. Then we include the second most needy group and we constrain this subset to be also means non-decreasing (but it does not hinder redistribution to be means-decreasing for the second most needy subgroup) and so on. At the last step, the least needy group enclosure reconstitutes the whole population and we constrain redistribution to be mean preserving. This device seems quite appealing but the major drawback is that we build a fictive tax system which blurs the effects of the existing one. Therefore, our first option of creating a mean-preserving redistribution seems unrealistic, so another way of dealing with this problem would be to use restricted stochastic dominance.

As was suggested in section 2.4, we may limit our range of concern, so that we do not, for instance, concern ourselves with what happens beyond a certain income level or percentage of the population. On the dual approach, we may restrict the dominance criterion to requiring that the generalized Lorenz curve is superior up to, say, the 90 percent point. It would not then matter if the curves intersected beyond that point. Nor would it matter that the total income were reduced by the tax system, provided that there is an increase in total income for the group with which we are concerned. However, this restricted dominance condition could also be defined in terms of the dual approach (i.e. 95 percent) or of the primal approach (i.e. for incomes less than three times the mean for instance). Ex-ante, the cut-off level should not decrease with the need level, the needier, the higher the cut-off level, the most conservative option being a constant cut-off line regardless of the need level.

However, in practice, an ex-post analysis is possible by looking for empirical cut-off points. These are the points for which the differences, either between the cdfs or between the integrals of the cdfs, cross the zero threshold (i.e. above this point FOD or SOD is not verified any longer). This cut-off line reveals the way the tax system implicitly values the different need groups. At each step of the SSD, would we restrain the income distribution of the whole population be low this cut-off point, that we would have SOD verified up to this subpopulation (i.e. up to this sequential step). Therefore, at the final step of SSD, the last cut-off point is the income threshold above which OSOD will never be verified for the whole population. In the light of SOD definition, those cut-off points reveal the income levels below which the total post-tax income is equal to the total pre-tax income for the restricted subpopulation (i.e. in terms of general Lorenz dominance, this is equivalent to the necessary condition that the post-tax mean has to be at least above the pre-tax mean for any dominance condition to be possible). This points out one of the main drawbacks of SD: the difficulty of using SD to assess the redistributive power of tax systems is that total post-tax income is always lower that total pre-tax income thus making SD hardly performing (or at least in its most general version). Nonetheless, the main advantage of our approach is that we do not modify the tax systems, which would lead to hardly interpretable results concerning the initial tax systems (for instance a very attractive idea would be of applying the French tax system to the German income distribution and compare the two post-tax distributions. However, this is hardly feasible in practice because both systems are too different and most of the time the differences do not originate, for instance, from the tax schedules but rather from the taxable income especially in the French-German comparison).

These cut-off points reveal how a tax system values the different need groups. In inter-country comparisons, the level of the cut-off line is less important than its shape (and more precisely than its slope). We will come back on this point in the next section with the comparison of the French and German tax systems.

2.4 *An application to France and Germany*

This section gives an example of how assessing the redistributive effects of the French and German tax systems and how to make a comparison of their respective redistributive power.

2.4.1 The data

For the empirical analysis for France, we use data from the French `Taxable Income Survey' for the year 2001 with 75000 private households. The data are provided by the French IRS (the `DGI, Direction Générale des Impôts'). The pre-tax household income is the gross income, including pensions, unemployment benefits and social contributions. The post-tax income is the pre-tax income less social contributions, income tax and after non means-tested child benefits (`allocations familiales') as well as means-tested benefits (`complément familial'). This post-tax income was chosen to study the family aspect of redistribution.

For the German side of the study, we use data from the 2002 wave of the German socio-economic panel study (GSOEP) which concerns therefore the 2001 incomes. It is a representative panel study of private households living in Germany. In 2002, there were about 12000 households in the survey. The pre-tax household income which we use in our survey, differs from the usual income measures of the German tax law. `pre-tax' household income used here is the sum of earnings from dependent employment and from self-employment, capital income, income from rent and lease as well as the full amount of these benefits in the pre-tax income. The post tax income is the pre-tax income less income tax and including child benefits.

2.4.2 Descriptive results

General descriptive results will be available in Baclet, Dell and Wrohlich (2007). The first step is to compute dominance stochastic of first and second order for the whole distribution between pre-tax and post-tax distributions. As foreseen, FOSD and SOSD hold neither for France nor for Germany because the means of pre-tax distributions are higher than those for the post-tax distributions (see figure 5 in the case of Germany).

2.4.3 Family ordering

The second step is to compute sequential stochastic dominance. We rank households types according to their decreasing needs in the following way: couples with four or more children, couples with three children, singles with two children or more, couples with two children, couples with one child, childless couples, singles with one child, singles. Obviously, the ranking of households with respect to their needs is very subjective, especially concerning the single parents families, because we should take into account the difficulties of raising a family with only one parent. The ranking of childless couple and single parent with one child is for instance very controversial. However, the results should be robust to legitimate changes in ordering. Figure 6 (resp. figure 7) presents the results for Germany for FOD (resp. SOD).

2.4.4 Restricted sequential Stochastic Dominance

As was stated above, OFOD and SOSD are not verified. Following an idea of Atkinson and Bourguignon (1987), who argue that full dominance is not necessarily required, and that the social planner could only bother about dominance ``up to the n^{th} percentile", we choose to observe for various household size in the two countries, for which percentiles the social planner actually does not bother (or does not seem to bother). These points, above which dominance is not assured anymore, are called (empirical) cut-off points. We call the series of points, plotted against household type the ``empirical cut-off line". We first provide a formal grounding for this concept, in the general framework of stochastic dominance. We then dwell on its interpretation in terms of underlying welfare functions.

We follow the notations used in Atkinson and Bourguignon1987. We thus have:

$$\forall y\ \forall i = 1, ...n,\ U_y^i(y) = \sum_{k=i}^{n} \varepsilon_k(y)$$
$$\text{where } \varepsilon_k(y) \geq 0 \ \ \forall k\ \forall y$$

$\varepsilon_n(y)$ is the social marginal value of income for group *n*, $\varepsilon_n + \varepsilon_{n-1}$ is that of the next, and so on. Need groups are ordered following a decreasing need magnitude: group number one has the highest social marginal valuation of income and is the neediest group, then comes group number two and so on. Under these assumptions, Atkinson and Bourguignon (1987) demonstrate that a necessary and sufficient condition for a distribution *f* to dominate another distribution f^* was that $\sum_{k=1}^{n} p_k \Delta\varphi^k(y) \leq 0 \ \ \forall y$.

However, these assumptions concern the whole distribution. Atkinson and Bourguignon (1989) suggested that the range of concerns could/should be restricted. For instance, we could choose to ignore what happens beyond a certain income level or in an upper quantile of the population.

To formalize this approach in a general but simple way, we will assume that for each need group, the social planner does decides not to be concerned by households whose income is above a certain threshold. This means in terms of preferences that the social planner will not take into account any possible welfare gain provided by an income rise; this assumption is consistent with the assumption of decreasing social marginal utility of income. at the limit, we can assume that the marginal utility of income above a certain level should be zero (i.e. utility is constant above this threshold). We constrain the former class of utility functions to take into account this new feature:

$$\forall k,\ \varepsilon_k(y) \geq 0\ \forall y \leq Z_k \ \textbf{and}\ \varepsilon_k(y) = 0\ \forall y > Z_k \quad [\textbf{A1restr.}]$$

Let further assume like in Atkinson and Bourguignon (1987) that:

$$\forall k, \frac{\partial \varepsilon_k(y)}{\partial y} \leq 0 \ \forall y \quad [\mathbf{A2.}]$$

With the chosen ordering of need groups, we need to have $Z_1 \geq Z_2 \geq ... \geq Z_n$. Indeed, social marginal utility of income should be increasing with needs i.e. $U_y^1(y) \geq U_y^2(y) \geq ... \geq U_y^n(y) \ \ \forall y$. In particular $0 = U_y^1(Z_1) \geq U_y^2(Z_1)$ and since $U_y^2(Z_2) = 0$ and $U_y^2(\cdot)$ is decreasing $Z_2 \leq Z_1$.

Now the social utility for group k is constant above the threshold Z_k and therefore any change in the household income above the limit will not affect the global welfare:

$$0 \leq U_y^k(Z_k) = \sum_{j=k}^{n} \varepsilon_j(Z_k) \leq \sum_{j=k}^{n} \varepsilon_j(Z_j) = 0$$

because $\forall j \geq k$, $\varepsilon_j(\cdot)$ being decreasing (A2), and $Z_j \leq Z_k$, $\varepsilon_j(Z_k) \leq \varepsilon_j(Z_j)$. Atkinson and Bourguignon (1987) established (p. 359, (12.16)) that the difference in social welfare could be written:

$$\Delta\mathbb{W} = -\sum_{k=1}^{n} p_k U_y^k(a) \Delta\varphi^k(a) + \int_0^a \sum_{k=1}^{n} p_k U_{yy}^k(y) \Delta\varphi^k(y) dy$$

With *a* being the upper bound of the support of the overall income distribution $a \geq Z_1$. With the further assumption that $\varepsilon_k(y) = 0 \ \forall y \geq Z_k$, it appears that a sufficient condition under (A1restr.) and (A2) for a distribution to dominate the other one is that

$$\sum_{k=1}^{i} p_k \Delta\varphi^k(y) \mathbf{1}_{y \leq Z_i}(y) \leq 0 \ \forall y \ \ \forall i = 1, ..., n \quad \textbf{[SOSeqRestr.Dom]}$$

Where $\varepsilon_j(Z_k)\Delta\varphi^k(y) = \Delta\int_0^y F^k(t)dt$. This sufficient condition generalizes in a very intuitive way the second order dominance condition of Atkinson and Bourguignon (1987) to a setting of restricted dominance.

Demonstration of the sufficient condition: let us assume first that $a > Z_1$ Then since $U_y^i(a) = 0 \ \forall i$

$$\Delta\mathbb{W} = \int_0^a \sum_{k=1}^{n} p_k U_{yy}^k(y) \Delta\varphi^k(y) dy$$

then following Atkinson and Bourguignon (1987), differentiating **(A1restr.)** and substituting we obtain:

$$\Delta\mathbb{W} \geq 0 \Leftrightarrow \int_0^a \left[\varepsilon_n'(y) \sum_{i=1}^{n} p_i \Delta\varphi^i(y) + \varepsilon_{n-1}'(y) \sum_{i=1}^{n-1} p_i \Delta\varphi^i(y) + ... \right.$$
$$\left. ... + \varepsilon_2'(y)(p_1\Delta\varphi^1(y) + p_2\Delta\varphi^2(y)) + \varepsilon_1'(y) p_1 \Delta\varphi^1(y) \right] dy \geq 0$$

Suppose now that (SOSeqRestr.Dom) is verified. For $y > Z_1$, $\varepsilon_1' \ (y) = 0$. For $y \le Z_1$, the last term of the expression above is non negative given (SOSeqRestr.Dom) for *i*=1. Let's look now at the second term from the right. For $y > Z_2$, it is zero. For $y \le Z_2$, (SOSeqRestr.Dom) for *i*=2 assures that $p_1 \Delta\varphi^1(y) I_{y \le Z_2(y)} + p_2 \Delta\varphi^2(y) I_{y \le Z_2(y)} \le 0$ and therefore the second term is non negative. And so on for the following terms.

Suppose now that $\exists i / Z_i \ge a$, then $\forall j \le i,\ Z_j \ge a$ and we need to check that (SOSeqRestr.Dom) leads to $\sum_{k=1}^{i} p_k U_y^k(a) \Delta\varphi^k(a) \le 0$; but since $\forall j \le i,\ Z_j \ge a$, we are back in a setting of unrestricted dominance, and (SOSeqRestr.Dom) implies $\sum_{k=1}^{i} p_k \Delta\varphi^k(a) \le 0,\ \forall j = 1,...,i$ as (12,17) in Atkinson and Bourguignon (1987)

Demonstration of the necessary condition:
The next step consists in establishing the necessary part of the proposition. Following the method used in the appendix in Atkinson and Bourguignon (1987), the necessary condition holds (the demonstration is the same except that we have to restrain the range of the assumptions below the threshold, which is logical since we do not care of what happens above it). Therefore, for all utility function verifying the first assumption, a necessary and sufficient condition is $\forall i,\ \sum_{k=1}^{i} p_k \Delta\varphi^k(y) \le 0,\ \forall y \le Z_i$.

The proof heavily draws on Chambaz and Maurin (1996). Let's first recall the two lemmas (first one in a discrete setting; second one in a continuous setting) used by Chambaz and Maurin (1996).

Lemma 1: Let $I = [0, Z]$ be an interval, *V* the set of continuous functions over *I*, and V^+ (resp. V^- the set of non-positive (resp. non-negative) continuous functions over *I*. Now let $(\omega_1,...,\omega_n)$ be a set of continuous functions over *I*, (i.e. the set belongs to V^n).

$$\sum_{k=1}^{n} w_k(y) U_k(y) \in V^+ \ \forall U_1(\cdot), ..., U_n(\cdot) \in V^- \iff w_i(\cdot) \in V^-, \forall i$$

Subsequently, we will use in the proof the fact that if the functions $(\omega_1,...,\omega_n)$ are not constantly negative over *I* then there exists $(U_1,...,U_n)$ belonging to V^- such that $\sum_{k=1}^{n} \omega_k(y) U_k(y) < 0,\ \forall y \in I$. The underlying intuition is to overweight the positive parts of the ω_i .

Lemma 2: Using the same notation as for Lemma 1, let *f* now be a continuous function over the interval *I*, giving

$$\int_0^a f(y) u(y) dy \ge 0, \forall u \in V^+ \iff f \in V^+$$

Using the expression from the previous section we have

$$\Delta\mathbb{W} = -\sum_{k=1}^{n} \varepsilon_k(a) D_k(a) + \int_0^a \sum_{k=1}^{n} \varepsilon'_k(y) D_k(y) dy$$

$$\text{where } D_k(y) = \sum_{j=1}^{k} p_j \Delta\varphi^j(y)$$

We demonstrate the **necessary condition** by demonstrating the [contraposée]. We assume that (SOSeqRestr.Dom) is not verified and we want to exhibit utility functions which verify (A1restr.) and (A2) and for which $\Delta W < 0$.

If (SOSSeqRestr.Dom) is not verified then there exists an integer *i* such that D_i is not constantly non-positive over $[0, Z_i]$. We choose the smaller value of integer *i* that verifies this assumption. From Lemma 1, we know there exists an interval $I \subset [0, Z_i]$ and a set $(\eta_1,...,\eta_n)$ of non-positive continuous functions over $[0, Z_i]$ such that:

$$\sum_{k=1}^{i} \eta_k(y) D_k(y) < 0, \forall y \in I$$

If we choose furthermore that $\eta_{i+1} = ... = \eta_n = 0$ (so that we do not take into account the groups whose needs are below group *i*), then we have:

$$\sum_{k=1}^{n} \eta_k(y) D_k(y) < 0, \forall y \in I$$

Lemma 2 informs us of the necessary existence of a non-negative continuous function *u* over *I* such that:

$$-\int_{\mathbf{I}} u(y) \sum_{k=1}^{n} \eta_k(y) D_k(y) dy > 0$$

By extending *u* over $[0, Z_i]$ (for instance by extending the function by 0 over the remaining part of $[0, Z_i]$ except at the upper and lower bounds of *I* where we have to enforce the continuity), we know there exists a non-negative continuous function such *u* such that

$$-\int_0^{Z_i} u(y) \sum_{k=1}^{n} \eta_k(y) D_k(y) dy > 0$$

If we define ε_k by :

$$(\varepsilon_k(y) = \int_0^y u(x)\eta_k(x)dx - \int_0^{Z_k} u(x)\eta(k)dx, \forall y \leq Z_k) \text{ and } (\varepsilon_k(y) = 0, \forall y \geq Z_k)$$

Then we have $\varepsilon'_k = u\eta$ and therefore $\varepsilon'_k \leq 0$. As ε_k is decreasing and $\varepsilon_k(Z_k) = 0$, $\varepsilon_k \geq 0$. Now consider a planner whose preferences can be infer from $(\varepsilon_1,...,\varepsilon_n)$. By construction, these preferences satisfy (A1restr.) and (A2) and we have

$$\Delta\mathbb{W} = \int_0^a u(y) \sum_{k=1}^{n} \eta_k D_k(y) dy < 0 \quad \text{qed.}$$

which concludes the proof.

This class of functions reveals the preferences of the social planner. In this case, he would not take into account the households whose incomes are above the thresholds to assess the redistributive effects of a tax system.

Since the assessment of the redistributive effects of a tax schedule is a very puzzling problem, this restrictive dominance provides an interesting framework for some international comparisons. Estimating the thresholds line reveals the implicit valuation of each needy group according to their income. For each needy group, the income threshold represents the income above which a household welfare variation should not matter. If we restrict ourselves to utility functions that are constant for the household whose income exceeds these thresholds, then we can state that the overall welfare is increasing. Therefore, the higher the thresholds are, the more households will the social planner have in his preferences.

2.4.5 Cut-off points and implicit equivalence scales

The previous section states that a series of cut-off points, increasing with need level, bounds the social planner preferences. Therefore Z_n represents the upper limit of income above which a single should not enter the scope of concern of the social planner. Z_{n-1} will represent the threshold for the second less needy group and so on.

The ratio Z_{n-1}/Z_n constitutes an equivalence scale which relates the two thresholds of the two least needy groups. The higher the ratio will be, the more significant the difference of concern of the planner will be. We can compute the ratio Z_i/Z_n at each step, those ratios relate the threshold of the different need groups to the one of the least needy group (most of the time the singles). To state that this tax schedule is welfare enhancing, is equivalent to assuming that the planner does not take into account the households above the thresholds. Therefore the higher those ratios are, the more sensitive the planner will be, to a need group, relative to the one immediately following in the ranking. Nonetheless, the progressivity of these ratios are highly dependent of the threshold for the least needy group Z_n.

As a conclusion, the empirical cut-off lines raise the veil on the implicit preferences of a tax schedule. They can be interpreted as an implicit equivalence scale in terms of preferences. The classical scales assess the progressivity of needs according to different criteria (subjective, consumption patterns...). In our case, the scale assesses the progressivity in the range of preferences of the social planner. Whereas most of the time, international comparisons concern theoretical schedules, this preferences scale, which is a more abstract one enables the comparison of actual tax schedules. Unfortunately, it can only be estimated at the point where marginal social utility of income goes to zero in the social welfare function: it therefore scratches a lot of information.

If the tax systems in France and Germany truly reflect the preferences of the social planner with regard to inequality, the cut-off lines reflect where social marginal utility of income becomes zero, for various groups.

2.4.6 Application

In order to quantify the respective redistributive effects of the French and the German system, we systematically assess the dominance of the after-tax distribution over the pre-tax distribution in both countries separately. Obviously, no complete dominance of the after-tax distribution over the pre-tax distribution can be achieved, be it in Germany or in France. We therefore focus on the empirical cut-off lines. In the French case, OSOD would be verified provided that we would restrain the population to its 17 % fractile, which is equivalent, in terms of income level, to a household income below 11900 euros. Would we not bother of the singles, then OSOD would be verified provided that we would restrain the population to have an income below 20000 euros which is equivalent to the bottom 39 % of the population. And so on for each group (figure 3).

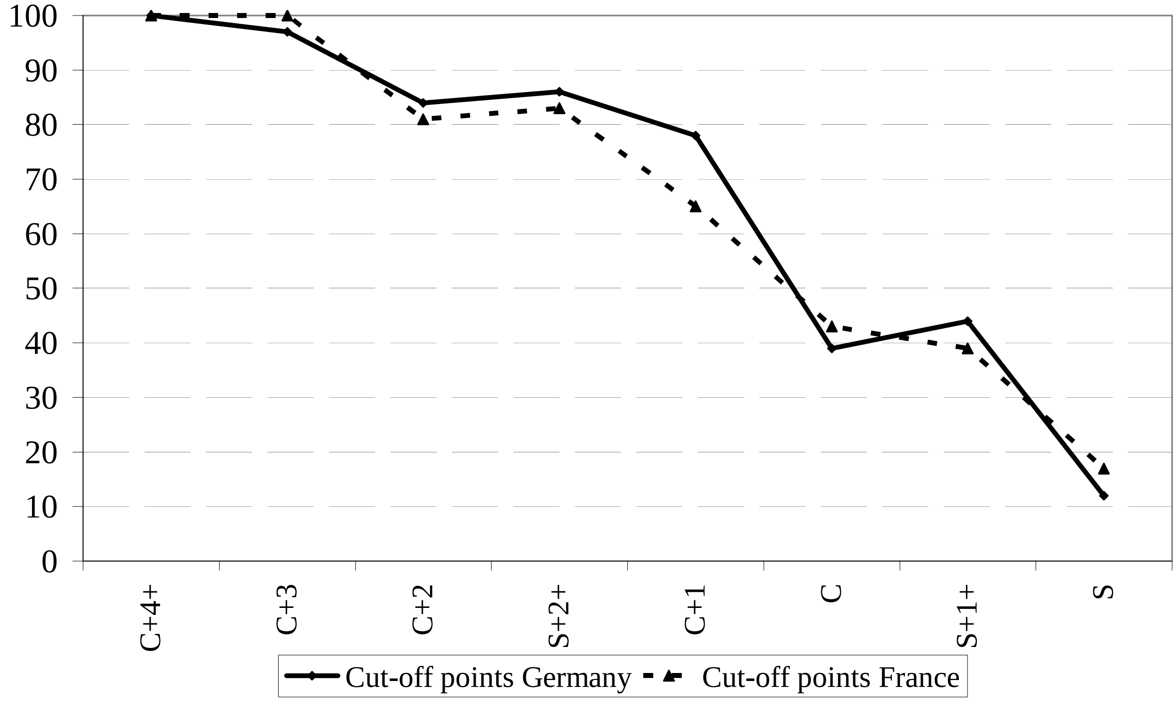


Figure 3: Empirical dominance cut-off line, Germany and France, 2001.
Sources: Calculations of the author GSOEP2002 and ERF2001.

3 The SAS macros

3.1 *Different SAS macros an hand*

The SAS macros are divided in four parts. Precise documentation of the functioning of each macro is to be found directly in the source code of the macros, which can be obtained under the following address mailto:-dg75-f350@insee.fr**.**

3.1.1 Overall Toolbox

``Overall Toolbox" contains the following macros: **%creationCdf**, **%creationCdf2**, **%creationCdf3**, **%graphs**, **%graphs2**, **%graphs3**, **%graphRedistribution** which are needed by the macros of the other parts.

3.1.2 Lorenz Dominance

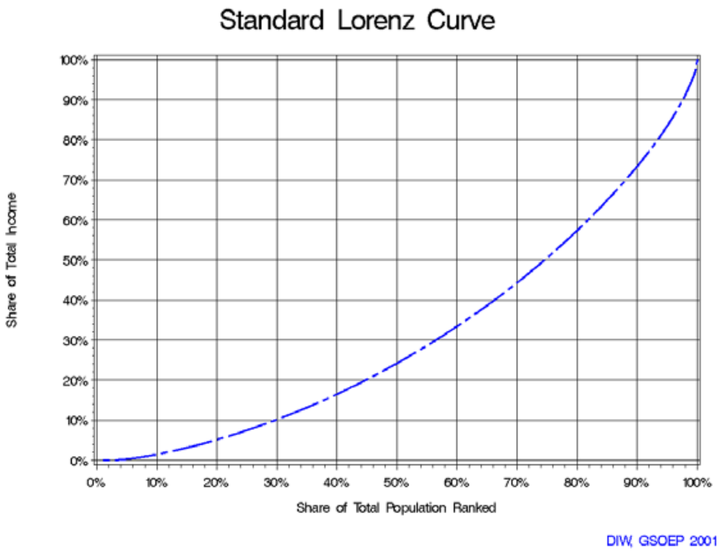
``Lorenz Dominance" contains the following macros: **%SLorenz**, **%SLorenzCurve**, **%GLorenz**, **%GLorenzCurve**, **%SLorenz2**, **%GLorenz2** which compute the classical Lorenz curves, and compare them. Examples of output produced by these macros are given in Table 3 and 4.

3.1.3 Stochastic Dominance

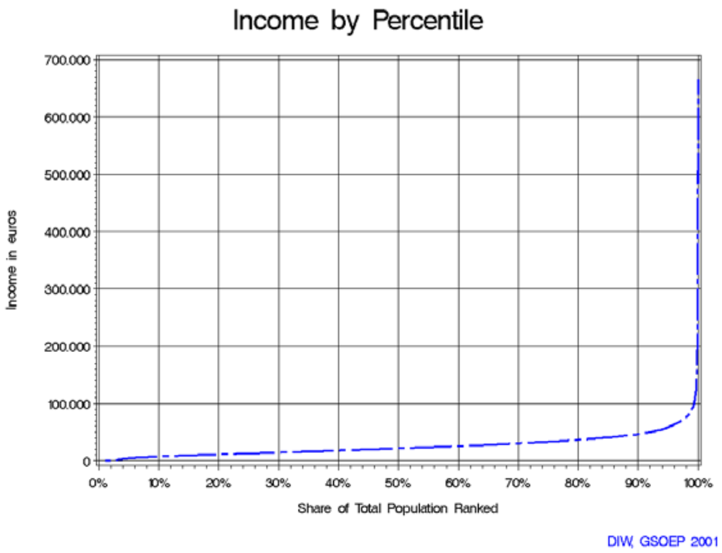
``Stochastic Dominances" contains the macro **%dominance** which computes and graphs first and second order stochastic dominance. Examples of output produced by this macro are given in Table 5.

3.1.4 Sequential Stochastic Dominance

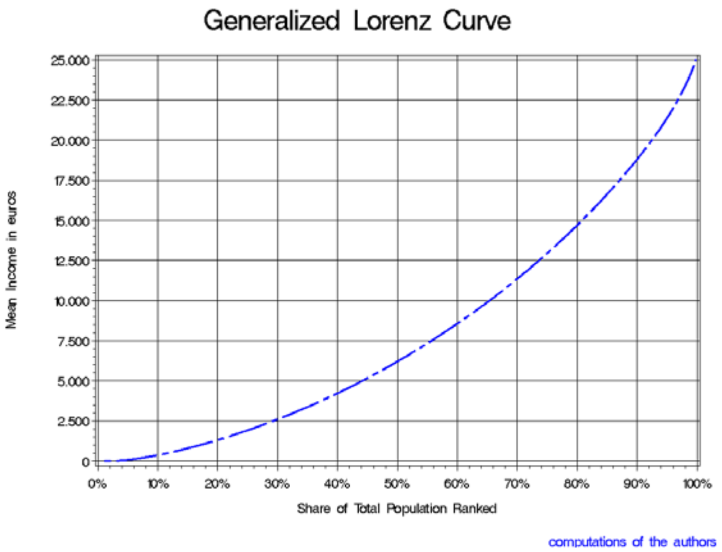
``Sequential Stochastic Dominances" contains the following macros: **%prepareTable**, **%sequentialDominance**, **%creationSeqBefore**, **%creationSeqAfter**, **%final**, **%findRestr2nd**, **%interNeedsRedistribution**, which compute and graphes first and second order sequential stochastic dominances. Examples of output produced by these macros are given in Tables 6 and 7.



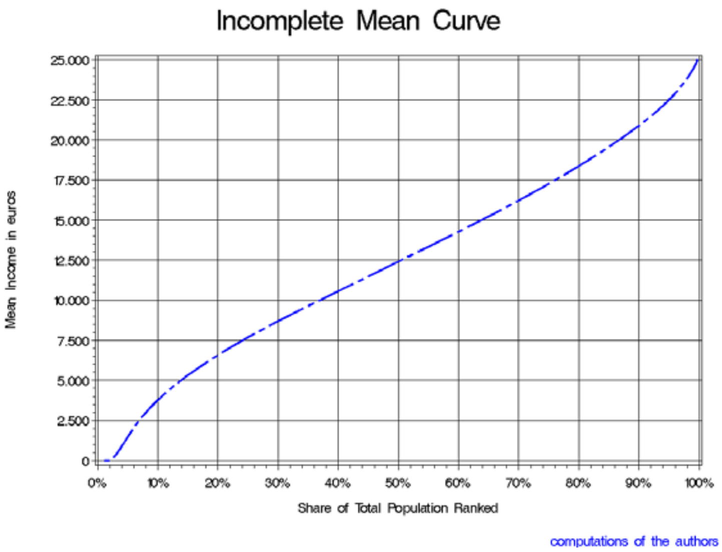
(a) Lorenz



(b) Income by percentile

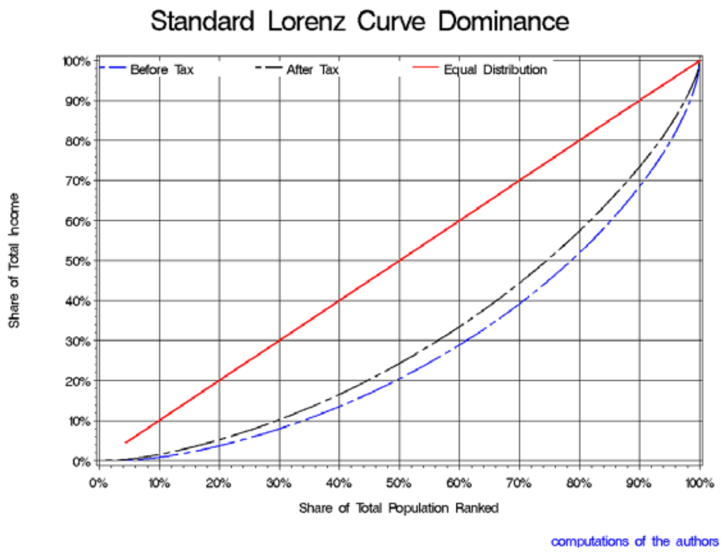


(c) Generalized Lorenz

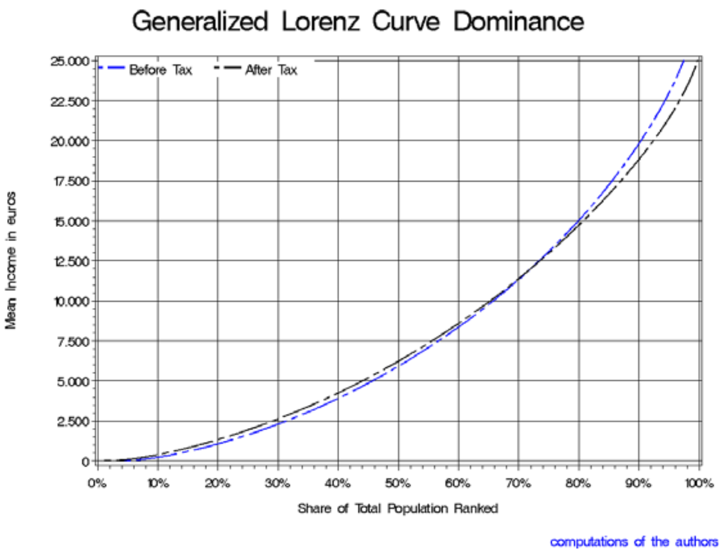


(d) IMC

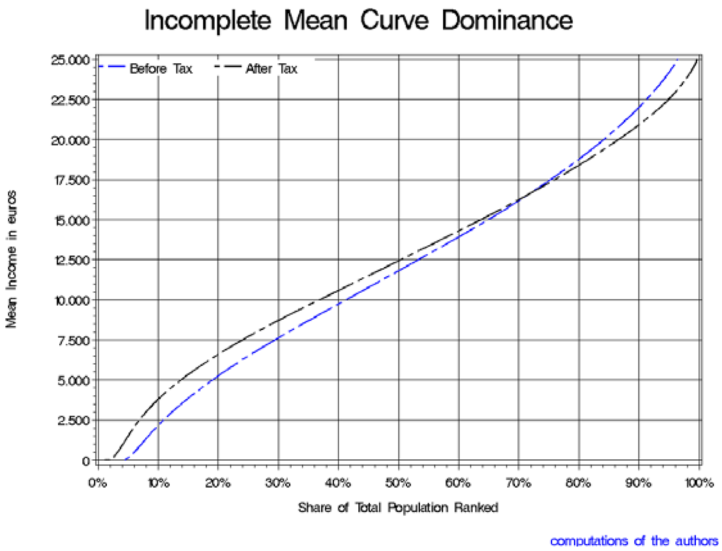
Figure 4: Lorenz Curves



(a) LD

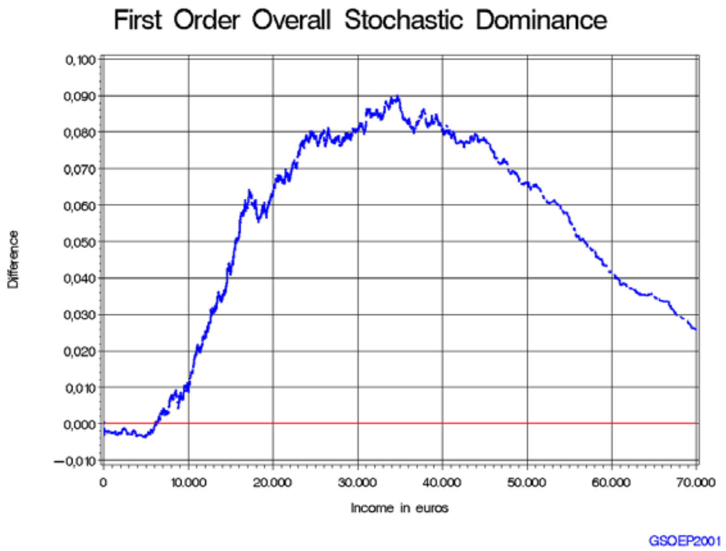


(b) GLD

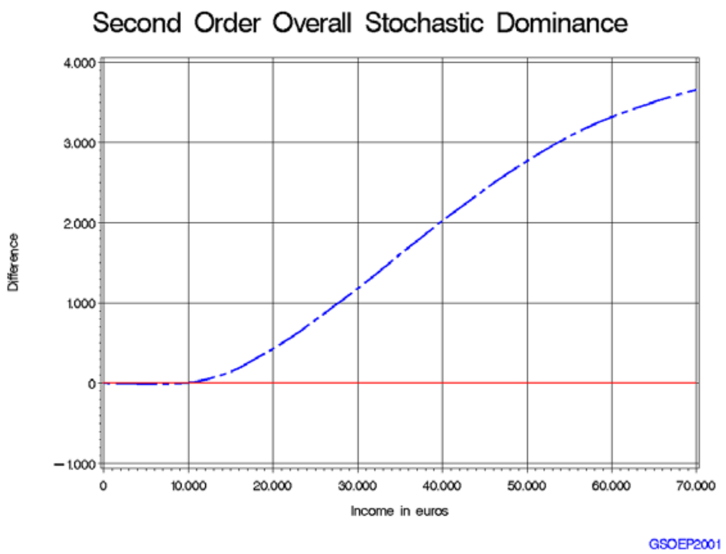


(c) IMC

Figure 5: Lorenz Dominances

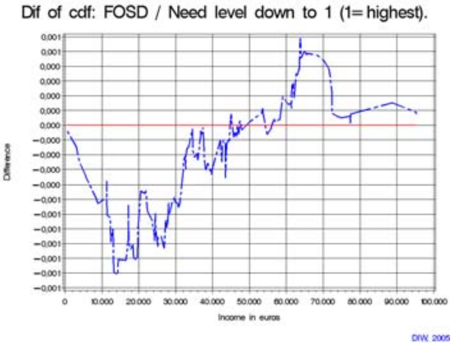


(a) FOD

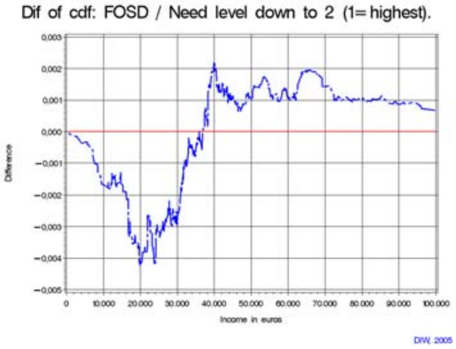


(b) SOD

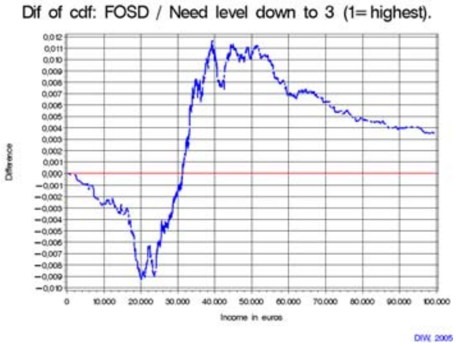
Figure 6: Overall Stochastic Dominances



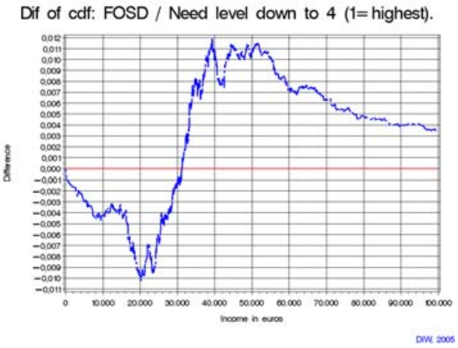
(a) Needs = 1



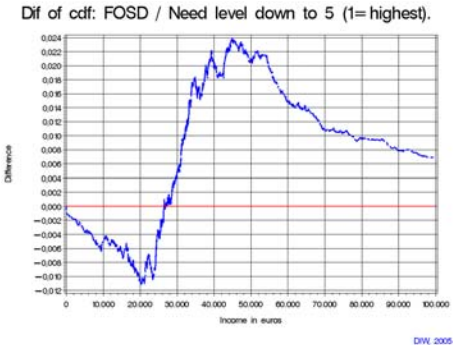
(b) Needs = 1-2



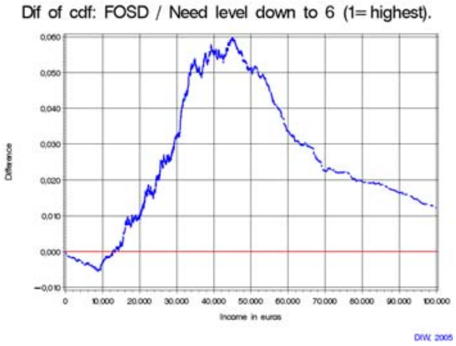
(c) Needs = 1-3



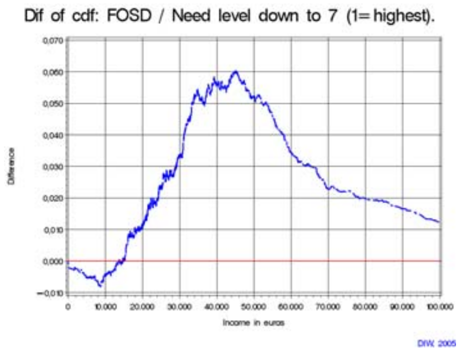
(d) Needs = 1-4



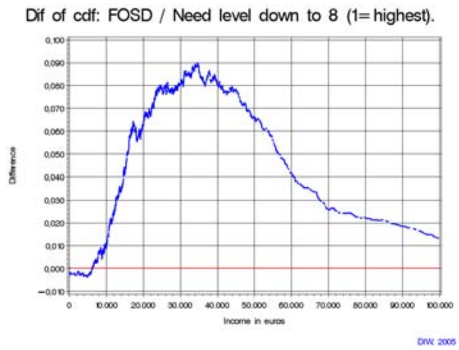
(e) Needs = 1-5



(f) Needs = 1-6

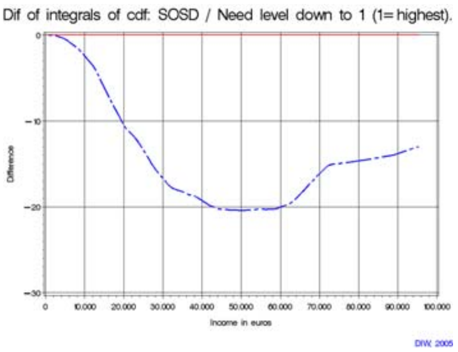


(g) Needs = 1-7

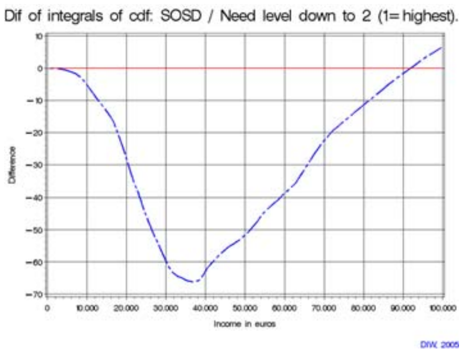


(h) Needs = 1-8

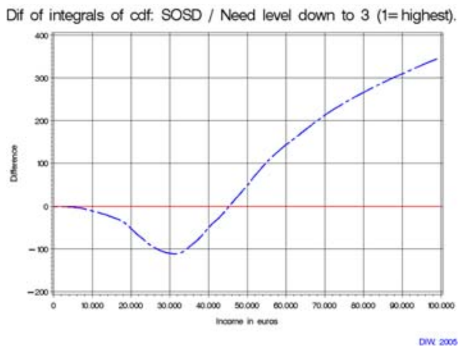
Figure 7: Sequential First Order Stochastic Dominance



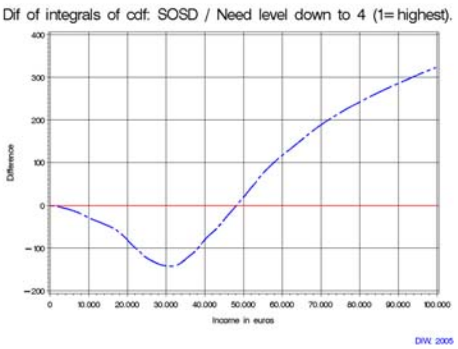
(a) Needs = 1



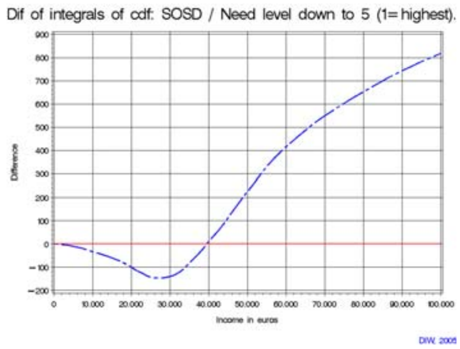
(b) Needs = 1-2



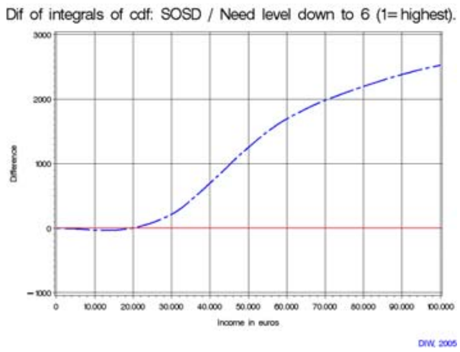
(c) Needs = 1-3



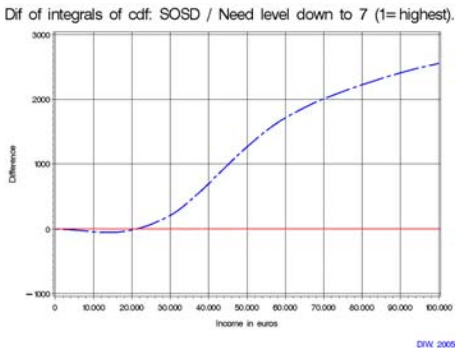
(d) Needs = 1-4



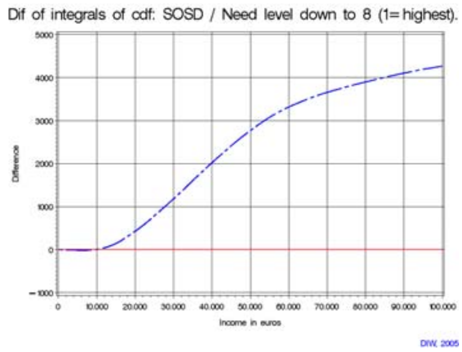
(e) Needs = 1-5



(f) Needs = 1-6



(g) Needs = 1-7



(h) Needs = 1-8. Overall second order dominance

Figure 8: Sequential Second Order Stochastic Dominance

3.2 Example of application

```
/* Use of Set 1 */
%SLorenz(table=germanyexample, library=work, variable=aftertax_hh, pond=hhrfk);
%SLorenz(table=germanyexample, variable=aftertax_hh, pond=hhrfk);
%SLorenzCurve(source="DIW, GSOEP 2001");
%GLorenz(table=germanyexample, variable=aftertax_hh, pond=hhrfk);
%GLorenzCurve (top=25000, step=2500);
%SLorenz2(table=germanyexample, variable1=pretax_hh,variable2=aftertax_hh,
pond=hhrfk, seuil=1, label1="Before Tax", label2="After Tax") ;

%GLorenz2(table=germanyexample,variable1=pretax_hh,variable2=aftertax_hh,
pond=hhrfk, seuil=1, label1="Before Tax", label2="After Tax", top=25000,
step=2500);


/* Use of Set 2 */
%dominance (variable1=pretax_hh , ponderation1=hhrfk , table1=germanyexample,
variable2=aftertax_hh , ponderation2=hhrfk, table2=germanyexample, librairie2=
work, pass_source="GSOEP2001", pass_top1 = 100000, pass_top2= 100000 );


%dominance (variable1=pretax_hh, ponderation1=hhrfk , table1=germanyexample,
variable2=aftertax_hh, pass_source="GSOEP2001", pass_top1 = 70000, pass_top2=
70000 );


/* Use of set 3 */
%prepareTable (table=germanyexample, librairy=work, work_librairy=work,
needs_classes=8, needs= needsshort);

%sequentialDominance(needs_classes=8,need_var=needsshort,
table=germanyexamplenomissing, library=work, variable1=pretax_hh,
variable2=aftertax_hh, pond=hhrfk);

%creation_seq_before(needs_classes=8, library=work, income=pretax_hh);
%creation_seq_after(needs_classes=8, library=work, income=aftertax_hh);
%final(needs_classes=8, library=work, income1=pretax_hh, income2=aftertax_hh);
%graphs2(needs_classes=8, library=work, top=100000, source="DIW, 2005");
%graphs3(needs_classes=8, library=work, top=10000, source="DIW, 2005");
%findRestr2nd (library=work, file="C:\WorkSpace\HBS\Stops.xls",
needs_classes=8,
library_perc_ref=work, data_perc_ref=germanyexample, var_perc_ref=pretax_hh,
weights_perc_ref=hhrfk, precision=1);

%interNeedsRedistribution(library=work,                    table=germanyexample,
variable1=pretax_hh,
variable2=aftertax_hh, weights=hhrfk, needs_classes=8, needs=needsshort);
```

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N°F0707

SEQUENTIAL STOCHASTIC DOMINANCE, PRINCIPLES AND IMPLEMENTATION. AN APPLICATION TO THE ASSESSMENT OF THE FRENCH AND GERMAN TAX SYSTEMS

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Juin 2007

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Abstract
This paper presents the principles of stochastic dominance which is a powerful tool that enables to have a generalized approach for the measure of poverty and inequalities.
This paper first summarizes the literature on stochastic dominance with all its latest developments. We then present how to apply stochastic dominance to the assessment of the redistribution of a tax system and discuss of the constraints that are bound to it. A tax system is designed to address equity concerns, but two dimensions have to be taken into account. The first one is the redistribution between high income and low income (vertical redistribution) and the second one is the redistribution from small families towards large families (horizontal redistribution). Therefore, we study and discuss the uses of sequential stochastic dominance (Atkinson and Bourguignon (1989)) and restricted dominance (Chambaz and Maurin (1998)) that seem well suited to the assessment of a tax system.
We finally provide SAS macros with the computations of all the stochastic dominances that are contained in the paper. We also provide an example of how to use the macros to compare the tax systems between France and Germany.

Key Words: Stochastic dominance, redistribution, tax system, inequality.

Résumé
Ce document de travail présente les principes et l’implémentation de la dominance stochastique, un puissant outil permettant une approche générale de la mesure des inégalités et de la pauvreté.
Ce papier résume dans une première partie la littérature sur la dominance stochastique et ses développements les plus récents. Nous présentons ensuite comment appliquer la dominance stochastique pour évaluer le caractère redistributif d’un système fiscal et discutons des contraintes que cela pose. Il est en effet important de distinguer deux dimensions dans un système fiscal. La première dimension concerne la redistribution verticale, c'est-à-dire la redistribution des ménages à hauts revenus en faveur des ménages à bas revenus. La dimensions horizontale concerne la redistribution en faveur des ménages de grande taille. Dans cette perspective, nous étudions et discutons de l’utilisation de la dominance stochastique séquentielle (introduite par Atkinson et Bourguignon (1989)) et de la dominance stochastique restreinte (voir Chambaz et Maurin (1998)) qui semblent être particulièrement bien adaptées à l’évaluation d’un système fiscal.
Finalement, nous mettons à disposition des macros SAS qui permettent de calculer l’ensemble des dominances stochastiques présentées dans ce document de travail. Nous proposons également un exemple d’utilisation dans le cadre d’une comparaison entre la France et l’Allemagne.

Mots clés : dominance stochastique, redistribution, taxation, inégalités

Introduction

Income is most of the time known at the household level. Measuring income inequalities ignoring differences among households along other dimensions than mean income (such as household composition) leads to highly distorted pictures, especially for inter-temporal or cross-country comparisons.

The use of equivalence scales enables one to introduce household composition in the study of income inequality. Indeed, income per adult-equivalent, an improved version of per capita income, can take accurately into account the economies of scale realized when individuals live together. For instance the French and German National Statistical Offices as well as Eurostat use the `modified OECD equivalence scale' which leads to divide the income of a couple with two children by $1 + 0,5 + 2 \times 0,3 = 2,1$ rather than by 4. This approach entails a number of problems which one may want to set aside. The theoretical grounds on which equivalence scales rely are still disputed (see Lechêne (1993)) and the empirical determination of the scale itself is far from being straightforward and often relies on additional hypotheses (See for instance Hourriez and Olier (1997) for an estimation in the case of France). The use of a uniform equivalence scale (with weights constant throughout the distribution) is for instance obviously controversial as it relies on normative assumptions, which are most of the time note openly revealed. But most of all, the parsimony principle is violated when one uses equivalence scales to compare income distributions on populations of heterogeneous households: comparisons can be made with less restrictive hypotheses. Sequential stochastic dominance provides the framework for such comparisons. It relies on a bivariate approach: utility is not a function of the sole (possibly per adult equivalized) income, but it is a function of both income (increasing) and needs of the household (decreasing). The needs of the household are, most of the time, proxied by its composition[2].

Another justification of the stochastic dominance approach is the following: as soon as studying income distribution in the end leads to public policy choices (like proposals for tax-benefit reforms), one would like the choice to be agreed upon by a wide range of individuals with possibly differing preferences with regard to redistribution. These differing preferences can be summarized within differing social welfare functions (swf). And the reforms can then be evaluated if they lead to a better (distribution) outcome for a wide range of swfs. The swfs can just be related to income (univariate case / overall dominance) or be related to both income and needs (bivariate case sequential dominance).

Nevertheless equivalence scales rely on a lot of assumptions and one may want to relax these assumptions in order to take into account household composition in the most general possible way. This is what stochastic dominance enables one to do.

1 Stochastic dominance: a survey of the basic result

[2] Note that other factors of need, which are of interest for social choice theory like invalidity for instance, can be taken into account following this approach

In this section, we first present the basic (univariate) setting for (overall) stochastic dominance. We then introduce heterogeneity among households and define sequential stochastic dominance. Lastly we present some relevant extensions.

1.1 Overall dominance

Let y be the continuous income variable over a bounded range $[0, y_{\max}]$, with probability density function (henceforth *pdf*) f and cumulative distribution function (henceforth *cdf*) F. Following Atkinson (1970), comparisons of inequality levels between two distributions can rely on social welfare functions (swf)[3]. In the simple case of symmetric and additively separable functions, they can be written:

$$\mathbb{W} = \int_0^{y_{max}} u(y) f(y) dy$$

Overall First Order Stochastic Dominance (OFOD) states that a distribution is preferred (meaning that it leads to a greater social welfare) to another if and only if the cdf of the preferred distribution is never above and at least once strictly beneath the other distribution. In order for this criterion to hold, the utility function must be assumed non-decreasing $u'_y(y) \geq 0 \ \ \forall y$ **(P1)**. This can be written:

$$\boxed{[\text{OFOD :}] \text{ if } \mathbf{P1} \text{ then: } f \succ_{\mathbb{W}} f^* \Leftrightarrow \forall y \; F(y) \leq F^*(y)}$$

Note that this approach can be extended to absolute monetary poverty: if the poverty line is defined exogenously, then the OFOD criterion is equivalent to saying that the poverty rate for the preferred distribution is lower, whatever the poverty line is.

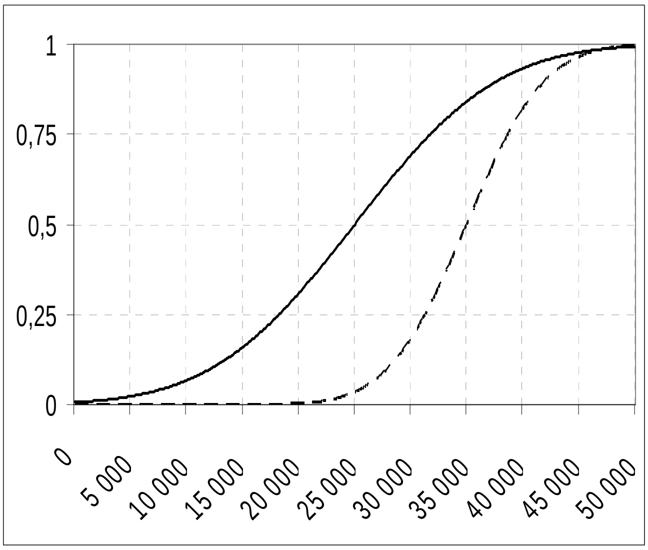
Unfortunately, in a wide number of cases --- as soon as the cdf cross each other --- the OFOD cannot conclude. Assuming slightly more restrictive characteristics of the utility function, a more powerful criterion can be derived, namely Overall Second Order Stochastic Dominance (OSOD). According to that criterion, a distribution is preferred to another if and only if the surface under the cdf of the preferred distribution is never greater and at least once strictly smaller than the surface under the other distribution. This criterion entails the additional assumption that, on the top of the utility function being non-decreasing, its first derivative has to be non-increasing $u''_y(y) \leq 0 \ \ \forall y$ **(P2)**. Intuitively, this criterion can deal with some crossing cdf. OSOD can be written as follows:

$$\boxed{[\text{OSOD :}] \text{ if } \mathbf{P1{,}P2} \text{ then: } f \succ_{\mathbb{W}} f^* \Leftrightarrow \forall y \int_0^y F(t)dt \leq \int_0^y F^*(t)dt}$$

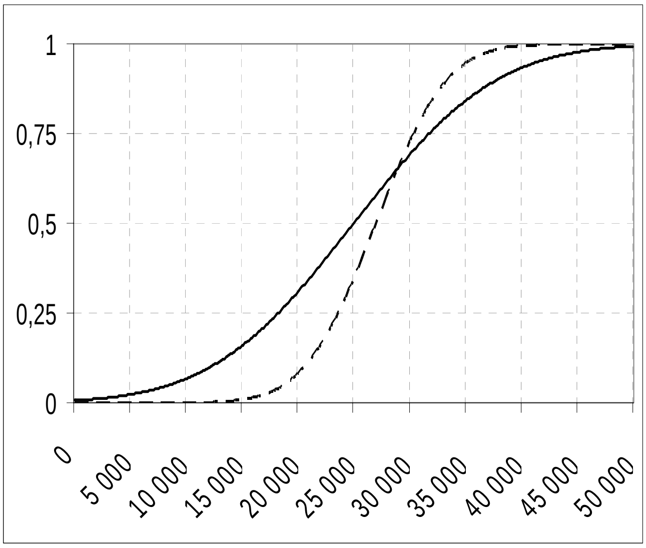
[3] Of course, a different approach leads to `summarize' the distribution with a real number (an `inequality index', which has to respect some normative properties) but the cost is a huge loss of information. The approaches presented here all lead to partial orders but with pretty consensual assumptions over the comparison criterion. In a classical trade-off setting, extending the subset of comparable distributions entails making ever more restrictive (ever less consensual) assumptions over the criterion.

Coming back to a poverty setting, OSOD means that the poverty gap of the preferred distribution is lower, whatever the poverty line is.

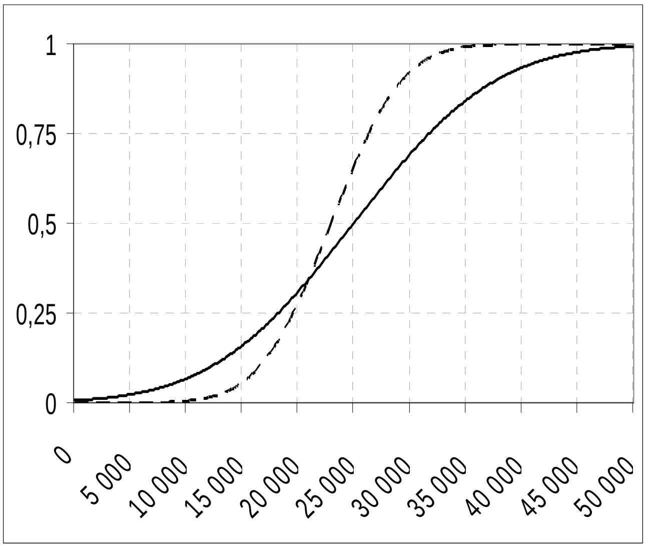
Foster and Shorrocks (1988) showed that OFOD was a sufficient but not necessary condition for higher order dominances. Higher order stochastic dominance criteria can be defined, and it can be shown (Davidson and Duclos (2000)), that *s* order stochastic dominance is equivalent



(a) FO



(b) No FO but SO



(c) Neither FO nor SO

Figure (1) Stochastic Dominance

to comparing Foster-Greer-Thorbecke P_{s-1} for all possible poverty lines. Figures 1(a) to 1(c) show the various situations possible.

[Figure 1abc]

1.2 Dominances and the Lorenz curve

The seminal presentation of Atkinson as been extended since Shorrocks (1983) introduced the equivalent concept of Generalized Lorenz Dominance (GLD) which consists in comparing the Lorenz curves, re-scaled by the mean income of the distribution. This concept is more intuitive than OD and explicitly addresses the trade-off between equity and efficiency underlying income distribution comparisons.

First, pure equity concerns are captured by requiring mean-preserving regressive transfers not to increase welfare. Following Dasgupta, Sen and Starrett (1973), this is equivalent to assuming the swfs to be Schur-concave[4] **(L1)**. In this setting, two distributions with equal means can be compared if their Lorenz curves do not intersect. The distribution corresponding to the Lorenz curve lying above is preferred. If the Lorenz curves intersect, no conclusion can be reached. Formally let L(**y**,p) be the Lorenz curve of the distribution **y**, with mean μ; $p \in [0,1]$. We then have:

$$\boxed{[\textbf{LD}:] \text{ if } \mu = \mu' \text{ and } \textbf{L1} \text{ then: } W(\mathrm{y}) \geq W(\mathrm{y}') \Leftrightarrow L(\mathrm{y}', p) \geq L(\mathrm{y}, p)\ \forall p}$$

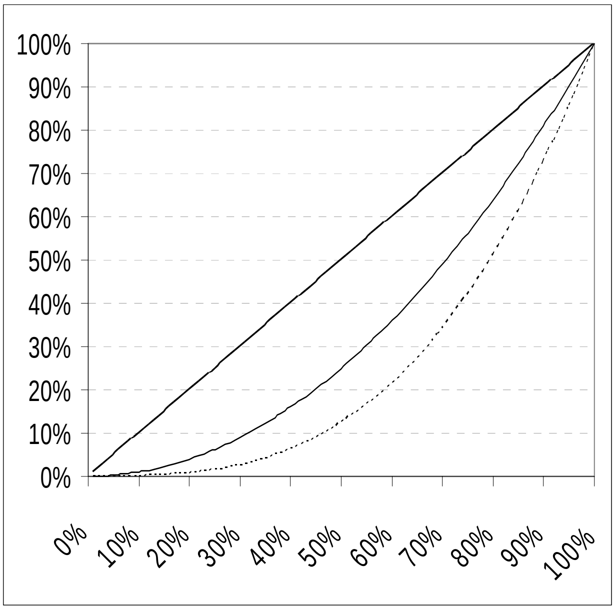
In order to introduce the efficiency dimension, we need to generalize this setting to the case where means differ. Indeed, higher mean income (efficiency) may possibly offset the impact of higher inequality (inequity) in the comparison. To take this aspect into account we have to further assume that the swf be increasing **(L2)**[5]. In this setting, two distributions with differing means can be compared if their generalized Lorenz curves do not intersect. The generalized Lorenz curve is the Lorenz curve re-scaled by the mean of the distribution. The distribution corresponding to the generalized Lorenz curve lying above is preferred. If the generalized Lorenz curves intersect, no conclusion can be reached. Let GL(**y**,p) be the generalized Lorenz curve of the distribution **y**, with mean μ; $p \in [0,1]$. We then have:

$$\boxed{[\textbf{GLD}:] \text{ if } \textbf{L1,L2} \text{ then: } W(\mathrm{y}) \geq W(\mathrm{y}') \Leftrightarrow GL(\mathrm{y}', p) \geq GL(\mathrm{y}, p)\ \forall p}$$

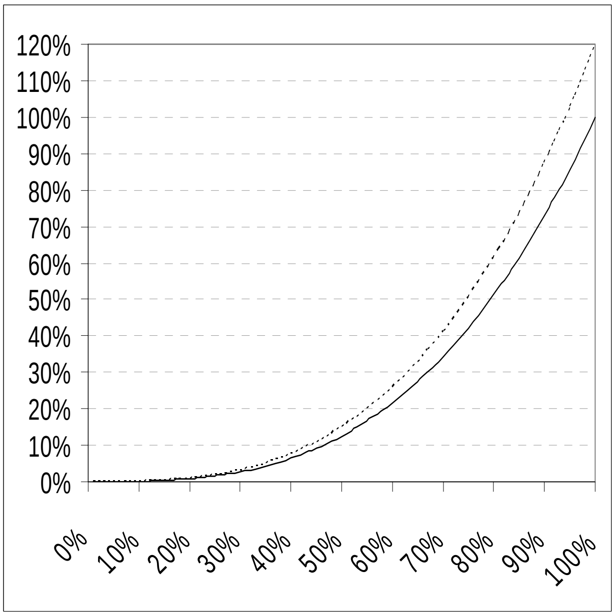
A sufficient condition for having GLD is when a distribution has both higher mean and higher (simple) Lorenz curve. Figure 2(a) and 2(b) illustrates how rescaling a more unequal distribution which has higher mean can lead it to GL-dominate although it was L-dominated. Figure 2(c) shows that even the most equal distribution is in the end GL-dominated by a distribution with slightly higher mean.

[4] Schur-concavity is a weak form of concavity. A function is Schur-concave if it translates the ordering of vectors according to majorization into the standard scalar ordering.
[5] Note that **L1, L2** are verified by utilitarian swf complying with **P1, P2**.

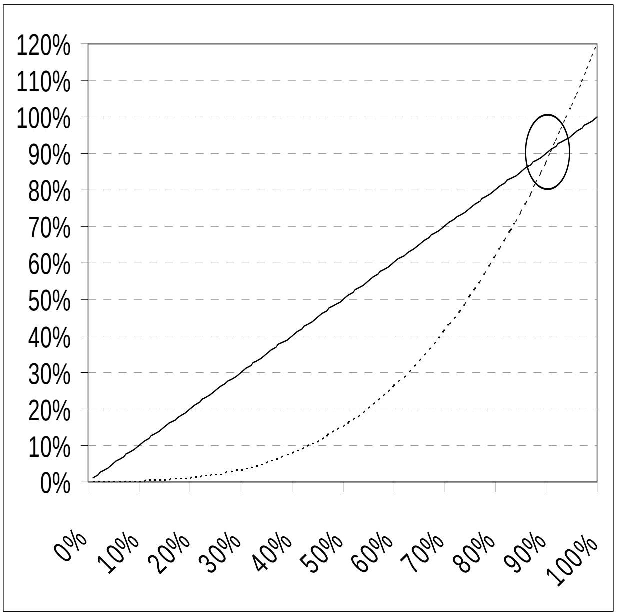
[Figure 2abc]



(b) LD



(b) GLD (1)



(c) GLD (2)

Figure (2) Lorenz Dominance

1.3 Dual approaches

The approaches in terms of Generalized Lorenz Dominance and in terms of Second Order Stochastic Dominance are dual. Following Atkinson and Bourguignon (1989), we present in this section this dualism with a common formalism.

1.3.1 "Primal approach" and the OSOD

The approach in which swf is defined in terms of ``expected utility"

$$W = \int_0^{y_{max}} u(y) f(y) dy$$

with $u' \geq 0$ and $u'' \leq 0$ leads to the following criterion

$$\Delta \int F(y)dy = \Delta \int_0^{y_{max}} \left(\int_0^y f(x)dx \right) dy \geq 0$$

1.1.1. "Dual approach" and the GLD

The criterion relying on GLD

$$\Delta \int_0^1 F^{-1}(p)dp = \Delta \int_0^{y_{max}} F^{-1}(F(y))dF(y) = \int_0^{y_{max}} y f(y) dy \geq 0$$

corresponds to an approach where swf is defined with respect to ranking:

$$W = \int_0^1 w(p) y(p) dp$$

where $w'' \leq 0$. It means that the weight w attached to the level y of income attained by individuals with relative rank p in the population is decreasing with the rank (and with the level of income since $p = F(y)$ with F increasing. Atkinson (1970) was the first to make the link between the two approaches.

1.3.2 Some trivial results to keep in mind

1. OFOD implies OSOD and therefore GLD

2. Comparing distributions with the same means, LD and GLD and, therefore OSOD are equivalent

3. When a distribution has a higher mean income, it cannot be GL-dominated by another distribution with lower mean income: more income can compensate more inequality, but the reverse does not hold (compare Figures 2(b) and 2(c)). Since GLD and OSOD are equivalent, and OSOD is necessary for OFOD to hold, when a distribution has a

higher man income, it can be neither OFO- not OSO-dominated by another distribution.

This latest simple result is of great importance for the practical implementation of stochastic dominance (see section 3).

1.4 Political economy concerns: robustness, consensus and "restricted dominance"

One of the advantages of the approach presented above is the low degree of specification of the swf underlying the criterion. As we have seen only S-concavity is requested, which is pretty general. What are the implications of this generality gain? We focus here on the comparison of distribution before and after redistribution (for the difficult assessment of inter-temporal or cross-country inequality variation, see section 3.2).

1.4.1 Ex-ante design of redistributive policies

Trying to design a tax-transfer system, and ignoring the real swf it is facing, the government may want to implement a reform implementing a GL-dominating distribution (either over the pre-tax-and-transfers distribution, or, as in Atkinson and Bourguignon (1989), over the actual post-tax-and-transfers distribution). Thus GLD helps to design reforms which are robust to our uncertainty over the real social preferences we are facing. Clearly, since GLD is a partial ordering only, the set of possible reforms is reduced by the desire of implementing a robust reform. This is the price to pay for robustness.

1.4.2 Ex-post evaluation

Evaluating ex-post either a reform, or a tax-transfer system, the economist may want to formulate judgements being as general as possible with regard to the swf underlying the analysis. The price to pay here is the eventuality for the analyst not to be able judge the reform or the system.

1.5 Consensus

Now clearly the generality over the swf models the fact that we ignore the real swf, not that there may be heterogeneity within the population. In promoting a GL-dominating reform, one can be sure that the next government with differing views (trying to implement a reform enhancing a welfare measured according to another swf) will still agree on his or her reform being welfare enhancing[6].

Nevertheless, if people can vote on my reform and have different views on the inequalities, the present framework does not help to conclude whether or not they will support the reform.

[6] We do not here engage in trying to address the tricky issue of cardinal measurement of the welfare gain and restrict ourselves to ordinal concerns.

To keep things simple we remain in a utilitarian %framework with symmetric, additively separable swf.

One way of constraining the criterion to gain comparability consists in writing the integrals of the criteria up to a given level say $\overline{y}$ (or in the dual approach up to a given percentile $\overline{y} = F(\overline{y})$) and to speak of dominance "up to $\overline{y}$". This can be called **``restricted dominance''** (see Atkinson and Bourguignon (1989)). If a reform or a tax-transfer system leads to dominating distribution ``up to the last decile'' for instance, the top decile of the population above the threshold is simply ignored in the evaluation. Ignoring in first approximation possible re-ranking across the threshold between the two distributions, one can thus state that the reform planned is ``supported'' by any well behaved swf of the bottom 90% of the population. The top decile may have different views, this will not be taken into account. This approach thus for instance allows for some heterogeneity of the individual utilities within an additive swf.

Conversely, a system which only leads to a dominating distribution ``up to the fourth decile'' is clearly not robust in case a democratic vote is cast upon it. One cannot exclude that the effective swf in the population leads more than 50 % individuals to vote for it, but there is at least one swf (and it may be the ``true one'') for which a 60 % block of the population may vote against the project.

Note that using restricted dominance criteria also loosen the necessary condition imposed on the mean income of the dominating distribution: only the mean income of the bottom x\% of the dominating distribution has now to be higher. William Gates may be taxed heavily and the tax revenue burnt, it will not affect the dominance ``up to William Gate'' (assuming Mr. Gate remain the richest individual, since our swfs obviously remain anonymous).

1.6 Sequential Dominance

We now come to income distributions on populations of heterogeneous households. We introduce a `needs' variable which is independent of the income variable. Atkinson and Bourguignon (1982} study the most general bi-continuous case. Atkinson and Bourguignon (1987) restrict themselves to discrete needs, which do not need to be cardinal. Bourguignon (1989) and Atkinson and Bourguignon (1989) implement this latter approach in the case where need is the size of the household. We quickly present both discrete approaches (qualitative and quantitative) approaches here.

Let the population be divided into n subsets generated by the `need' variable. Let p_i be the share of the i-th subset in the total population $\sum_{i=1}^{n} p_i = 1$. We suppose that the index i ranks the subsets by increasing need level (or equivalently by decreasing well being level, conditional on income, i can thus be thought of as the size of the family, for the sake of the intuition). Group n is the `most deserving group'. $u^i(y)$ denotes utility function of households of the subset i. Remaining within a setting of symmetric and additively separable swfs, we have:

$$\mathbb{W} = \int_0^{y_{max}} u(y)f(y)dy = \sum_{i=1}^{n} p_i \int_0^{y_{max}} u_i(y)f_i(y)dy$$

with $f_i(y) = f(i/y)$.

Now we need to make assumptions on how u^i varies with i. If no hypothesis is made (which means that we assume that no agreement *at all* can be reached within the population on the way u^i varies with i), then the only way to compare the different groups is to require dominance for each group taken separately. As Atkinson and Bourguignon (1989) put it "this is highly restrictive and precludes any redistribution between family types"[7].

Let us suppose now that it is possible to rank marginal (income) utilities according to the reverse ranking according to which (income) utilities are ranked. Intuitively, if the `need' variable is the size of the household, then it means that we assume that marginal (income) utilities are increasing with the size of the household. For instance, conditional on income, large families have a lower utility but a higher marginal utility than singles. This is quite natural if we assume that marginal utilities are non-increasing.

In the qualitative framework, this assumption (**P3**) can be written as follows:

$$\forall y\ \forall i = 1,...n,\ u_y^i(y) = \sum_{k=1}^{i} \varepsilon^k(y) \text{ où } \varepsilon^k(y) \geq 0\ \ \forall k\ \forall y$$

In the quantitative framework, we have simply:

$$\forall y\ \ u_y^i(y) \text{ non-decreasing with } i$$

First Order Sequential Dominance can then be written as:

$$\boxed{[\textbf{SFOD :}]\ \text{if } \mathbf{P1,P3} \text{ then: } f \succ_{\mathbb{W}} f^* \Leftrightarrow \left(\forall y\ \forall j = 1,...,n\ \sum_{i=j}^{n} p_i F(y) \leq \sum_{i=j}^{n} p_i F^*(y)\right)}$$

Practically it means that we successively verify the FOD on the increasing suite of subsets, beginning with the population with the higher needs (and which thus has, conditional on income, the higher social marginal utility) and adding step by step to this core population the sub-populations with lesser needs. The last FOD to be verified is obviously the OFOD. Therefore SFOD strictly implies the OFOD.

Like for the overall dominance criteria, some additional restrictive hypotheses on utility functions (i.e. limiting the range of the possible consensus on a `dominant' reform) lead to a more general dominance criterion (closer to a complete order). Let us suppose that the differences between two successive marginal (income) utilities are decreasing with `needs'.

[7] At the other extreme, using equivalence scales means that the way u^i varies with i is completely determined (or subject to consensus in the population), and that we can replace $u^i(y)$ by $u(y/e_i)$ where e_i is the scale.

In the qualitative framework, this assumption (P4) can be written as:

$$\forall y, \forall k = 1, ..., n \ \varepsilon_y^k(y) \leq 0$$

In the quantitative framework, we have simply:

$$\forall y \ \ u_{yy}^i(y) \text{ non-increasing with } i$$

Second Order Sequential Stochastic Dominance can then be written as:

$$\boxed{[\text{SSOD :}] \text{ if } \mathbf{P1,P2,P3,P4} \text{ then: } f \succ_W f^* \Leftrightarrow \left(\forall j = 1, ..., n \ \forall y \ \sum_{i=j}^{n} p_i \int_0^y F(t)dt \leq \sum_{i=j}^{n} p_i \int_0^y F^*(t)dt \right)}$$

We now focus on `needs' being defined as the size of the family. If we have **P1-P4** then we can write **P5**[8]:

$$u_y^{i+1}\left(\frac{i+1}{i}y\right) \leq u_y^i(y)$$

this means that `the marginal utility of income decreases with family size at constant income per capita'. We can now write another version of the SOSD:

$$[\text{SSOD}^* \text{ :}] \ f \succ_W f^* \Leftrightarrow \begin{array}{l} \sum_{i=1}^{n} p_i \int_0^{y_i} F(t)dt \leq \sum_{i=1}^{n} p_i \int_0^{y_i} F^*(t)dt \\ \forall y_i \text{ verifying } \frac{i}{i+1} y_{i+1} \leq y_i \leq y_{i+1} \end{array}$$

This, as shown in Fleurbaey et *al.* (2003) can lead back to an approach in terms of generalized equivalence scales.

1.7 Necessary condition for SSD to be verified

Since a higher mean income of the dominating distribution is a necessary condition for the criterion to be verified at each step of the sequential process, the average redistribution between household groups is constrained. As soon as the mean income of one of the sets decreases, the criterion fails. This implies first of all the neediest group cannot see its mean income decrease. Then, the adjunction of the second neediest group cannot lead the mean income of the reunion of the two groups to decrease. This means that, although the mean income of the second-worse off group may decrease, it should be at least offset by a rise of the mean income of the neediest group, in order for the average of the two to remain at least constant. And so on. The average inter-group redistribution thus has to take place top-down, from the less needy to the needier.

[8] See Bourguignon (1989), p.70.

1.8 Restricted SSD and the question of the cut-off point

The sequential dominance approach can also be restricted to gain comparability. The definition of a constant cut-off line across the household types is nevertheless not an easy one (see section 3 for more details).

1.9 Extension: Equivalence scales

P1 and **P2** are quite consensual hypotheses, but **P3** and **P4** are more likely to be controversial since they assume comparability of marginal utility levels across households.
One solution to relax these hypotheses is to make use of `fuzzy equivalence scales'. The basic idea is that a consensual comparison of two different households will always be reached if incomes are sufficiently different. For instance, compared to a single with \$1 000, a couple with \$1 000 will always be considered worse off and a couple with \$2 000 better off. Between these bounds however, no consensus might be reached. Nonetheless, the bounds constitute a structure which can be used to compare income distributions on heterogeneous populations: some intermediate point between `classical' (non-robust) equivalence scale and no equivalence scales at all.

Taking as a reference category \$i=1\$, the considerations above lead to the following formal setting:

$$\begin{array}{l} (\textbf{P3'a}):\ u_i'(\alpha_i y) \leq u_{i+1}'(y)\ \forall y \in \mathbb{R}_+,\ \forall i \in 2,...,n \\ (\textbf{P3'b}):\ u_i'(\beta_i y) \leq u_{i+1}'(y)\ \forall y \in \mathbb{R}_+,\ \forall i \in 2,...,n \end{array}$$

Note that **P3** is a particular case of **P3'a**, with $\alpha_i = 1$. Note that **P5** is a particular case of **P3'b** with $\beta_i = \dfrac{i}{i-1}$. Thus the set of utility functions considered in Bourguignon (1989) (defined by **P1-P5** which are implied by **P1-P3'a&b**) is a superset of the utility functions considered here, which are thus less general than those considered for the SSD. Fleurbaey *et al.* (2003) show that under these assumptions a generalized SSOD $\succ_{\alpha,\beta}$ can be written, which depends on the vectors $(\alpha)_i$ and $(\beta)_i$, in R^n.

The classical equivalence scale approach implies the definitions of a vector $e = (e_1,..., e_2)$ of deflators (often $e_1 = 1$). In this framework we can define the swf:

$$\mathbb{W} = \sum_{i=1}^{n} p_i e_i \int_0^{y_{max}} f_i(y) u\left(\frac{y}{e_i}\right) dy$$

with *u* verifying **P1** and **P2**. Obviously, another dominance can be defined in this setting, depending on *e*.

Let us now write:

$$\Theta(\alpha,\beta) = \{(e_1,...,e_n) | \forall i = 2,...,n,\ \alpha_i e_{i-1} \leq e_i \leq \beta_i e_{i-1}\}$$

it is then possible to define a dominance ($\succ_{\Theta(\alpha,\beta)}$) (namely over the whole set of e within $\Theta(\alpha,\beta)$). This amount to define an overall dominance uniformly over a set of equivalence scales --- the set depending on (α,β). Fleurbaey *et al.* (2003) show that:

$$\boxed{f \succ_{\Theta(\alpha,\beta)} f^* \Leftrightarrow f \succ_{\alpha,\beta} f^*}$$

Thus the two approaches (sequential dominance vs. equivalence scales) meet at this point where the Bourguignon (1989) setting is restricted (and the order is thus less partial) and the equivalence scale paradigm is weakened (the complete order is lost but some consensus over the ranking is obtained).

2 Advantages and pitfalls of using stochastic dominance

2.1 Measuring inequalities

2.1.1 Inequalities and equivalence scales

The measure of income inequalities is a two-dimension problem. The first one concerns households of different sizes and compositions and the second one regards pure income inequalities. It is now common practice to use equivalence scales to reduce the problem to one dimension thus making different households incomes comparable. The underlying rationale of equivalence scale is to take into account the economies of scales within large households. However there exist many different ways of choosing an equivalence scale. First, one can use a normative scale which is devised by experts such as the “modified OECD equivalence scale” now used by many countries. The second type of scales are the ones implicit in the social security system, either in the tax schedule (for instance the French family-splitting) or in the benefit transfers. The third kind of equivalence scales are the ones estimated from the household budget surveys which reveal the consumption patterns of households. Finally, the last method found in the literature is based on subjective welfare measurement. There exists therefore a wide range of methods to compute equivalence scale which lead to a wide range of results.

As was quoted by Atkinson and Bourguignon (1989}, a survey by Whiteford in 1985 tabulates 44 different estimates of a scale for a single person, ranging from 49 to 94 percent of the couple scale. For a couple with two children, he quotes 59 estimates varying from 111 percent of the scale for a couple to 193 percent. The geometric means of these estimates is 138 percent, but the OECD, for example, uses a figure of 159 percent. Regarding France, (Hourriez and Olier (1997)) assessed equivalence scales with different methods. Their concluding remarks spoke in favour of a revision of the standard Oxford scale used in those times. They argued that the Oxford scale was no longer suitable because it underestimated the economies of scale in the contemporaneous societies. The consumption patterns have evolved over the last decades, especially regarding the share of food in the household budget. Thus lower coefficients for each extra household member (i.e. higher economies of scale) would better fit reality. More recently, Koulovatianos et *al* (2005) showed that equivalence scales should be income dependent because the consumption economies of scale change as living standards go up. The intuition underlying this feature is that the income share dedicated to an extra member in the household decreases with the income level. Therefore, no consensus exists on the use of one particular equivalence scale.

2.1.2 from equivalence scales to stochastic dominance

When it comes to deal with inequalities, most papers use aggregate indicators such as the Gini, Atkinson or Theil indicators and each of them need equivalence scales to make every income comparable. Those indicators have two major drawbacks. First of all, they are unitless (i.e. independent of the level of the measured quantity). They solely measure the inequality of a given distribution. They are normative indicators and the lower the indicator, the lower the inequalities. For instance, according to this kind of indicators, Slovakia has fewer inequalities than France. However, in this case, the levels of the two distributions matter. This is directly related to the trade-off between equity and efficiency (see section 2.3). Those indicators do not take efficiency into account, they only focus on equity. On the contrary, SD is not independent of the level of the measured quantity. If two distributions have different means, OSOD analyses the trade-off between efficiency and equity in terms of welfare enhancement (provided that SD gives an ordering), the second drawback concerns the use of equivalence scales. The weight attached to the welfare of children is likely to be a matter of social judgment and they are likely to differ. It is therefore preferable to adopt an approach that treats differences in social judgments in a parallel manner to the way that Lorenz dominance treats different distributional judgments in the income dimension. The use of sequential stochastic dominance (SSD) makes explicit the redistribution between different family types which may be concealed by the use of equivalence scales.

As a conclusion SD provides a more general framework than equivalence scales. Whereas standard indicators focus on inequalities, SD assesses welfare enhancement. In the next section, we will study the advantages and drawbacks of equivalence scales versus SD for inter-country and inter-temporal comparisons.

2.2 Inter-temporal and inter-country comparisons

2.2.1 Inter-temporal comparisons

Many studies focus on the evolution of inequalities over time, trying to determine whether they increase or decrease, in the long run or in the short run. The use of equivalence scales supposes that the underlying assumptions regarding their estimation method have not significantly changed. If it uses consumption patterns then they should be the same. If it is based on subjective welfare, the preferences should be the same. Over the short run, these assumptions seem reasonable, however, over the long run, they are more questionable as Hourriez and Olier (1998) pointed it out (see section 3.1.1).

SD assumes that the population can be divided in different categories that can be unambiguously ranked according to their needs. Thus dominance results are restricted to cases where the marginal distribution of needs is fixed in the distributions being compared.

Moreover, when dealing with inter-temporal comparisons, one has to take inflation into account. Obviously, when comparing two distributions across time, the past distribution needs to be actualized by the rate of inflation in order to have two comparable distributions in terms of real income. In France for instance, the real income mean is increasing in average. A According to the SD properties (see section 2.3), an increase in inequalities can be offset by an increase in the real income, leading to a welfare enhancement, whereas the converse does not hold. Therefore, any ordering based on the dominance criterion will be dependent of inflation. On the contrary, standard indicators using equivalence scales solely focus on inequalities regardless of the real income evolution.

2.2.2 Inter-country comparisons

The second question related to inequalities concerns inter-country comparisons. The choice of an official equivalence scale for inter-country comparisons seems very controversial. Each national equivalence scale has its own country specificity. Nonetheless a study by (De Vos and Zaida (1997)) showed that the ranking of countries of the European community with respect to overall poverty is hardly affected by the use of different equivalent scales. However, and that seems to be a strong pattern of equivalence scales, it leads to huge differences in terms of poverty composition. Nevertheless, these results concern European countries which are rather homogenous. If we extend such comparisons to other countries, the choice of equivalence scale seems to affect poverty, inequality and therefore the ranking of countries according to those indicators (Buhmann et *al* (1988)).

How does SD perform in inter-country comparisons? Using SD to determine which distribution dominates the other one faces too many restrictions. First, inter-countries comparisons raise the problems of purchasing power parity. The exchange rate between the two countries can fluctuate and therefore lead to controversial results from one year to the next. Tackling this problem is possible, by restraining this kind of study to European countries for instance (Chambaz and Maurin (1998)). Similarly to inter-temporal comparisons, inter-country comparisons raise the problem of needs ordering as well as the means of the income distributions. Once more if one of the distributions has a higher mean then, if a ranking is possible, it will probably dominate the other one even if it is more unequal.

Finally, one of the most important problems raised by inter-country comparisons concerns income definition. It is actually difficult to have a homogenous income definition among different countries which is essential for a comparison to be consistent.

The restrictions facing SD make it unsuited to inter-temporal and inter-country comparisons. Standard indicators with equivalence scales perform better, unfortunately the results are not robust to a change in the scale choice.

2.3 Assessing the impact of tax systems

2.3.1 Stochastic dominance and redistributions

The tax system is designed to address equity concerns, but two dimensions have to be taken into account. The first one concerns the redistribution between high income and low income (i.e. vertical redistribution), and the second one the redistribution from small families towards large families (i.e. horizontal redistribution). As was pointed out in the previous section, the use of equivalence scales precludes any assessment of horizontal redistribution. On the contrary, all the restrictions concerning stochastic dominance make it more suited to assessing the redistributive effects of taxes and benefits at a given time period. In particular, sequential dominance enables one to assess explicitly the horizontal component of redistribution whereas OFOD and OSOD address solely vertical redistribution. Usually, the need ordering is given by household size even if the case of single-parent families needs some care. In the next section, we give some advices on the good practices that anyone should keep in mind while dealing with stochastic dominance and redistribution.

2.3.2 Good practices

Even if stochastic dominance seems to be well suited to assess the redistributive effect of the tax system, there are some constraints to care about. First, a constant pattern of tax systems is a decrease between the means of the pre-tax and post-tax distributions. As a matter of fact apart of its redistributive function, the tax system is also meant to produce tax revenue for government expenditures. Therefore, as was pointed out in the second part, even if the second distribution is the most equal possible (i.e. if everyone has the same post tax income), it will never dominate the pre-tax distribution. As we will see in the next section, that is what we obtain whether in France or Germany. More generally OFOD and OSOD will never be verified up to the top of the distribution. OFOD will fail because there is inevitably an income level above which the post-tax income is below the pre-tax income and therefore the two cdf cross. OSOD will also fail because the mean of the post-tax distribution will be below the mean of the pre-tax distribution. Therefore, it leaves us with two options, either to find a device to make the redistribution mean preserving, or to use restricted dominance (see section 2.5).

Our first option is to create a mean-preserving redistribution. The first idea would be to re-scale the post-tax distribution to its pre-tax level. This is equivalent to Lorenz dominance. However, in such a case, the post-tax distribution dominates the pre-tax distribution, which is a logical result as long as the tax schedule is progressive. This result simply states that the post-tax distribution is less unequal than the pre-tax distribution. Therefore this device does not help to compare the redistributive properties of two tax schedules.

A second idea would be to compensate the post-tax income by adding a lump sum transfer to each household in order to have the same mean than the pre-tax distribution. The economic legitimacy of such a transfer would be that the tax revenue enables the government to enhance public services that benefit equally to he whole population. Obviously, the major drawback of this option is that the population does not, actually, equally share the benefits. Apart from this criticism, this assumption seems too strong because the post-tax distribution will mechanically dominate the post tax distribution, as long as the tax system is progressive (because a translation does not modify inequality).

A third idea would be to constrain the redistribution to be mean preserving for each need category. However this hinders any horizontal redistribution which is not realistic. A more realistic device would be to constrain redistribution to lead to non-decreasing means for the post-tax distributions for each embedded sequence of subset of the population. More precisely, we constrain redistribution for the more needy group to be means non-decreasing. Then we include the second most needy group and we constrain this subset to be also means non-decreasing (but it does not hinder redistribution to be means-decreasing for the second most needy subgroup) and so on. At the last step, the least needy group enclosure reconstitutes the whole population and we constrain redistribution to be mean preserving. This device seems quite appealing but the major drawback is that we build a fictive tax system which blurs the effects of the existing one. Therefore, our first option of creating a mean-preserving redistribution seems unrealistic, so another way of dealing with this problem would be to use restricted stochastic dominance.

As was suggested in section 2.4, we may limit our range of concern, so that we do not, for instance, concern ourselves with what happens beyond a certain income level or percentage of the population. On the dual approach, we may restrict the dominance criterion to requiring that the generalized Lorenz curve is superior up to, say, the 90 percent point. It would not then matter if the curves intersected beyond that point. Nor would it matter that the total income were reduced by the tax system, provided that there is an increase in total income for the group with which we are concerned. However, this restricted dominance condition could also be defined in terms of the dual approach (i.e. 95 percent) or of the primal approach (i.e. for incomes less than three times the mean for instance). Ex-ante, the cut-off level should not decrease with the need level, the needier, the higher the cut-off level, the most conservative option being a constant cut-off line regardless of the need level.

However, in practice, an ex-post analysis is possible by looking for empirical cut-off points. These are the points for which the differences, either between the cdfs or between the integrals of the cdfs, cross the zero threshold (i.e. above this point FOD or SOD is not verified any longer). This cut-off line reveals the way the tax system implicitly values the different need groups. At each step of the SSD, would we restrain the income distribution of the whole population be low this cut-off point, that we would have SOD verified up to this subpopulation (i.e. up to this sequential step). Therefore, at the final step of SSD, the last cut-off point is the income threshold above which OSOD will never be verified for the whole population. In the light of SOD definition, those cut-off points reveal the income levels below which the total post-tax income is equal to the total pre-tax income for the restricted subpopulation (i.e. in terms of general Lorenz dominance, this is equivalent to the necessary condition that the post-tax mean has to be at least above the pre-tax mean for any dominance condition to be possible). This points out one of the main drawbacks of SD: the difficulty of using SD to assess the redistributive power of tax systems is that total post-tax income is always lower that total pre-tax income thus making SD hardly performing (or at least in its most general version). Nonetheless, the main advantage of our approach is that we do not modify the tax systems, which would lead to hardly interpretable results concerning the initial tax systems (for instance a very attractive idea would be of applying the French tax system to the German income distribution and compare the two post-tax distributions. However, this is hardly feasible in practice because both systems are too different and most of the time the differences do not originate, for instance, from the tax schedules but rather from the taxable income especially in the French-German comparison).

These cut-off points reveal how a tax system values the different need groups. In inter-country comparisons, the level of the cut-off line is less important than its shape (and more precisely than its slope). We will come back on this point in the next section with the comparison of the French and German tax systems.

2.4 An application to France and Germany

This section gives an example of how assessing the redistributive effects of the French and German tax systems and how to make a comparison of their respective redistributive power.

2.4.1 The data

For the empirical analysis for France, we use data from the French `Taxable Income Survey' for the year 2001 with 75000 private households. The data are provided by the French IRS (the `DGI, Direction Générale des Impôts'). The pre-tax household income is the gross income, including pensions, unemployment benefits and social contributions. The post-tax income is the pre-tax income less social contributions, income tax and after non means-tested child benefits (`allocations familiales') as well as means-tested benefits (`complément familial'). This post-tax income was chosen to study the family aspect of redistribution.

For the German side of the study, we use data from the 2002 wave of the German socio-economic panel study (GSOEP) which concerns therefore the 2001 incomes. It is a representative panel study of private households living in Germany. In 2002, there were about 12000 households in the survey. The pre-tax household income which we use in our survey, differs from the usual income measures of the German tax law. `pre-tax' household income used here is the sum of earnings from dependent employment and from self-employment, capital income, income from rent and lease as well as the full amount of these benefits in the pre-tax income. The post tax income is the pre-tax income less income tax and including child benefits.

2.4.2 Descriptive results

General descriptive results will be available in Baclet, Dell and Wrohlich (2007). The first step is to compute dominance stochastic of first and second order for the whole distribution between pre-tax and post-tax distributions. As foreseen, FOSD and SOSD hold neither for France nor for Germany because the means of pre-tax distributions are higher than those for the post-tax distributions (see figure 5 in the case of Germany).

2.4.3 Family ordering

The second step is to compute sequential stochastic dominance. We rank households types according to their decreasing needs in the following way: couples with four or more children, couples with three children, singles with two children or more, couples with two children, couples with one child, childless couples, singles with one child, singles. Obviously, the ranking of households with respect to their needs is very subjective, especially concerning the single parents families, because we should take into account the difficulties of raising a family with only one parent. The ranking of childless couple and single parent with one child is for instance very controversial. However, the results should be robust to legitimate changes in ordering. Figure 6 (resp. figure 7) presents the results for Germany for FOD (resp. SOD).

2.4.4 Restricted sequential Stochastic Dominance

As was stated above, OFOD and SOSD are not verified. Following an idea of Atkinson and Bourguignon (1987), who argue that full dominance is not necessarily required, and that the social planner could only bother about dominance ``up to the n^{th} percentile", we choose to observe for various household size in the two countries, for which percentiles the social planner actually does not bother (or does not seem to bother). These points, above which dominance is not assured anymore, are called (empirical) cut-off points. We call the series of points, plotted against household type the ``empirical cut-off line". We first provide a formal grounding for this concept, in the general framework of stochastic dominance. We then dwell on its interpretation in terms of underlying welfare functions.

We follow the notations used in Atkinson and Bourguignon1987. We thus have:

$$\forall y \; \forall i = 1, ...n, \; U_y^i(y) = \sum_{k=i}^{n} \varepsilon_k(y)$$
$$\text{where } \varepsilon_k(y) \geq 0 \;\; \forall k \; \forall y$$

$\varepsilon_n(y)$ is the social marginal value of income for group n, $\varepsilon_n + \varepsilon_{n-1}$ is that of the next, and so on. Need groups are ordered following a decreasing need magnitude: group number one has the highest social marginal valuation of income and is the neediest group, then comes group number two and so on. Under these assumptions, Atkinson and Bourguignon (1987) demonstrate that a necessary and sufficient condition for a distribution f to dominate another distribution f^* was that $\sum_{k=1}^{n} p_k \Delta\varphi^k(y) \leq 0 \;\; \forall y$.

However, these assumptions concern the whole distribution. Atkinson and Bourguignon (1989) suggested that the range of concerns could/should be restricted. For instance, we could choose to ignore what happens beyond a certain income level or in an upper quantile of the population.

To formalize this approach in a general but simple way, we will assume that for each need group, the social planner does decides not to be concerned by households whose income is above a certain threshold. This means in terms of preferences that the social planner will not take into account any possible welfare gain provided by an income rise; this assumption is consistent with the assumption of decreasing social marginal utility of income. at the limit, we can assume that the marginal utility of income above a certain level should be zero (i.e. utility is constant above this threshold). We constrain the former class of utility functions to take into account this new feature:

$$\forall k, \, \varepsilon_k(y) \geq 0 \; \forall y \leq Z_k \; \textbf{and} \; \varepsilon_k(y) = 0 \; \forall y > Z_k \;\; [\textbf{A1restr.}]$$

Let further assume like in Atkinson and Bourguignon (1987) that:

$$\forall k,\ \frac{\partial \varepsilon_k(y)}{\partial y} \le 0\ \forall y\ \ [\mathbf{A2.}]$$

With the chosen ordering of need groups, we need to have $Z_1 \ge Z_2 \ge ... \ge Z_n$. Indeed, social marginal utility of income should be increasing with needs i.e. $U_y^1(y) \ge U_y^2(y) \ge ... \ge U_y^n(y)\ \ \forall y$. In particular $0 = U_y^1(Z_1) \ge U_y^2(Z_1)$ and since $U_y^2(Z_2) = 0$ and $U_y^2(\cdot)$ is decreasing $Z_2 \le Z_1$.

Now the social utility for group k is constant above the threshold Z_k and therefore any change in the household income above the limit will not affect the global welfare:

$$0 \le U_y^k(Z_k) = \sum_{j=k}^{n} \varepsilon_j(Z_k) \le \sum_{j=k}^{n} \varepsilon_j(Z_j) = 0$$

because $\forall j \ge k,\ \varepsilon_j(\cdot)$ being decreasing (A2), and $Z_j \le Z_k,\ \varepsilon_j(Z_k) \le \varepsilon_j(Z_j)$. Atkinson and Bourguignon (1987) established (p. 359, (12.16)) that the difference in social welfare could be written:

$$\Delta \mathbb{W} = -\sum_{k=1}^{n} p_k U_y^k(a)\Delta\varphi^k(a) + \int_0^a \sum_{k=1}^{n} p_k U_{yy}^k(y)\Delta\varphi^k(y)dy$$

With *a* being the upper bound of the support of the overall income distribution $a \ge Z_1$. With the further assumption that $\varepsilon_k(y) = 0\ \forall y \ge Z_k$, it appears that a sufficient condition under (A1restr.) and (A2) for a distribution to dominate the other one is that

$$\sum_{k=1}^{i} p_k \Delta\varphi^k(y)\mathbf{1}_{y\le Z_i}(y) \le 0\ \forall y\ \ \forall i = 1,...,n\ \ [\mathbf{SOSeqRestr.Dom}]$$

Where $\varepsilon_j(Z_k)\Delta\varphi^k(y) = \Delta\int_0^y F^k(t)dt$. This sufficient condition generalizes in a very intuitive way the second order dominance condition of Atkinson and Bourguignon (1987) to a setting of restricted dominance.

Demonstration of the sufficient condition: let us assume first that $a > Z_1$ Then since $U_y^i(a) = 0\ \forall i$

$$\Delta \mathbb{W} = \int_0^a \sum_{k=1}^{n} p_k U_{yy}^k(y)\Delta\varphi^k(y)dy$$

then following Atkinson and Bourguignon (1987), differentiating **(A1restr.)** and substituting we obtain:

$$\Delta \mathbb{W} \ge 0 \Leftrightarrow \int_0^a \left[\varepsilon_n'(y)\sum_{i=1}^{n} p_i\Delta\varphi^i(y) + \varepsilon_{n-1}'(y)\sum_{i=1}^{n-1} p_i\Delta\varphi^i(y) + ...\right.$$

$$\left. ... + \varepsilon_2'(y)(p_1\Delta\varphi^1(y) + p_2\Delta\varphi^2(y)) + \varepsilon_1'(y)p_1\Delta\varphi^1(y)\right] dy \ge 0$$

Suppose now that (SOSeqRestr.Dom) is verified. For $y > Z_1$, $\varepsilon_1'(y) = 0$. For $y \leq Z_1$, the last term of the expression above is non negative given (SOSeqRestr.Dom) for *i*=1. Let's look now at the second term from the right. For $y > Z_2$, it is zero. For $y \leq Z_2$, (SOSeqRestr.Dom) for *i*=2 assures that $p_1 \Delta\varphi^1(y) I_{y \leq Z_2(y)} + p_2 \Delta\varphi^2(y) I_{y \leq Z_2(y)} \leq 0$ and therefore the second term is non negative. And so on for the following terms.

Suppose now that $\exists i / Z_i \geq a$, then $\forall j \leq i,\ Z_j \geq a$ and we need to check that (SOSeqRestr.Dom) leads to $\sum_{k=1}^{i} p_k U_y^k(a) \Delta\varphi^k(a) \leq 0$; but since $\forall j \leq i,\ Z_j \geq a$, we are back in a setting of unrestricted dominance, and (SOSeqRestr.Dom) implies $\sum_{k=1}^{i} p_k \Delta\varphi^k(a) \leq 0,\ \forall j = 1,...,i$ as (12,17) in Atkinson and Bourguignon (1987)

Demonstration of the necessary condition:
The next step consists in establishing the necessary part of the proposition. Following the method used in the appendix in Atkinson and Bourguignon (1987), the necessary condition holds (the demonstration is the same except that we have to restrain the range of the assumptions below the threshold, which is logical since we do not care of what happens above it). Therefore, for all utility function verifying the first assumption, a necessary and sufficient condition is $\forall i,\ \sum_{k=1}^{i} p_k \Delta\varphi^k(y) \leq 0,\ \forall y \leq Z_i$.

The proof heavily draws on Chambaz and Maurin (1996). Let's first recall the two lemmas (first one in a discrete setting; second one in a continuous setting) used by Chambaz and Maurin (1996).

Lemma 1: Let $I = [0, Z]$ be an interval, *V* the set of continuous functions over *I*, and V^+ (resp. V^- the set of non-positive (resp. non-negative) continuous functions over *I*. Now let $(\omega_1,...,\omega_n)$ be a set of continuous functions over *I*, (i.e. the set belongs to V^n).

$$\sum_{k=1}^{n} w_k(y) U_k(y) \in V^+ \ \forall\, U_1(\cdot), ..., U_n(\cdot) \in V^- \iff w_i(\cdot) \in V^-, \forall i$$

Subsequently, we will use in the proof the fact that if the functions $(\omega_1,...,\omega_n)$ are not constantly negative over *I* then there exists $(U_1,...,U_n)$ belonging to V^- such that $\sum_{k=1}^{n} \omega_k(y) U_k(y) < 0,\ \forall y \in I$. The underlying intuition is to overweight the positive parts of the ω_i.

Lemma 2: Using the same notation as for Lemma 1, let *f* now be a continuous function over the interval *I*, giving

$$\int_0^a f(y) u(y) dy \geq 0, \forall u \in V^+ \iff f \in V^+$$

Using the expression from the previous section we have

$$\Delta \mathbb{W} = -\sum_{k=1}^{n} \varepsilon_k(a) D_k(a) + \int_0^a \sum_{k=1}^{n} \varepsilon'_k(y) D_k(y) dy$$

$$\text{where } D_k(y) = \sum_{j=1}^{k} p_j \Delta\varphi^j(y)$$

We demonstrate the **necessary condition** by demonstrating the [contraposée]. We assume that (SOSeqRestr.Dom) is not verified and we want to exhibit utility functions which verify (A1restr.) and (A2) and for which $\Delta W < 0$.

If (SOSSeqRestr.Dom) is not verified then there exists an integer *i* such that D_i is not constantly non-positive over $[0, Z_i]$. We choose the smaller value of integer *i* that verifies this assumption. From Lemma 1, we know there exists an interval $I \subset [0, Z_i]$ and a set $(\eta_1, ..., \eta_n)$ of non-positive continuous functions over $[0, Z_i]$ such that:

$$\sum_{k=1}^{i} \eta_k(y) D_k(y) < 0, \forall y \in I$$

If we choose furthermore that $\eta_{i+1} = ... = \eta_n = 0$ (so that we do not take into account the groups whose needs are below group *i*), then we have:

$$\sum_{k=1}^{n} \eta_k(y) D_k(y) < 0, \forall y \in I$$

Lemma 2 informs us of the necessary existence of a non-negative continuous function *u* over *I* such that:

$$-\int_{\mathbf{I}} u(y) \sum_{k=1}^{n} \eta_k(y) D_k(y) dy > 0$$

By extending *u* over $[0, Z_i]$ (for instance by extending the function by 0 over the remaining part of $[0, Z_i]$ except at the upper and lower bounds of *I* where we have to enforce the continuity), we know there exists a non-negative continuous function such *u* such that

$$-\int_0^{Z_i} u(y) \sum_{k=1}^{n} \eta_k(y) D_k(y) dy > 0$$

If we define ε_k by :

$$(\varepsilon_k(y) = \int_0^y u(x)\eta_k(x)dx - \int_0^{Z_k} u(x)\eta(k)dx, \forall y \leq Z_k) \text{ and } (\varepsilon_k(y) = 0, \forall y \geq Z_k)$$

Then we have $\varepsilon'_k = u\eta$ and therefore $\varepsilon'_k \leq 0$. As ε_k is decreasing and $\varepsilon_k(Z_k) = 0$, $\varepsilon_k \geq 0$. Now consider a planner whose preferences can be infer from $(\varepsilon_1, ..., \varepsilon_n)$. By construction, these preferences satisfy (A1restr.) and (A2) and we have

$$\Delta \mathbb{W} = \int_0^a u(y) \sum_{k=1}^{n} \eta_k D_k(y) dy < 0 \quad \text{qed.}$$

which concludes the proof.

This class of functions reveals the preferences of the social planner. In this case, he would not take into account the households whose incomes are above the thresholds to assess the redistributive effects of a tax system.

Since the assessment of the redistributive effects of a tax schedule is a very puzzling problem, this restrictive dominance provides an interesting framework for some international comparisons. Estimating the thresholds line reveals the implicit valuation of each needy group according to their income. For each needy group, the income threshold represents the income above which a household welfare variation should not matter. If we restrict ourselves to utility functions that are constant for the household whose income exceeds these thresholds, then we can state that the overall welfare is increasing. Therefore, the higher the thresholds are, the more households will the social planner have in his preferences.

2.4.5 Cut-off points and implicit equivalence scales

The previous section states that a series of cut-off points, increasing with need level, bounds the social planner preferences. Therefore Z_n represents the upper limit of income above which a single should not enter the scope of concern of the social planner. Z_{n-1} will represent the threshold for the second less needy group and so on.

The ratio Z_{n-1}/Z_n constitutes an equivalence scale which relates the two thresholds of the two least needy groups. The higher the ratio will be, the more significant the difference of concern of the planner will be. We can compute the ratio Z_i/Z_n at each step, those ratios relate the threshold of the different need groups to the one of the least needy group (most of the time the singles). To state that this tax schedule is welfare enhancing, is equivalent to assuming that the planner does not take into account the households above the thresholds. Therefore the higher those ratios are, the more sensitive the planner will be, to a need group, relative to the one immediately following in the ranking. Nonetheless, the progressivity of these ratios are highly dependent of the threshold for the least needy group Z_n.

As a conclusion, the empirical cut-off lines raise the veil on the implicit preferences of a tax schedule. They can be interpreted as an implicit equivalence scale in terms of preferences. The classical scales assess the progressivity of needs according to different criteria (subjective, consumption patterns...). In our case, the scale assesses the progressivity in the range of preferences of the social planner. Whereas most of the time, international comparisons concern theoretical schedules, this preferences scale, which is a more abstract one enables the comparison of actual tax schedules. Unfortunately, it can only be estimated at the point where marginal social utility of income goes to zero in the social welfare function: it therefore scratches a lot of information.

If the tax systems in France and Germany truly reflect the preferences of the social planner with regard to inequality, the cut-off lines reflect where social marginal utility of income becomes zero, for various groups.

2.4.6 Application

In order to quantify the respective redistributive effects of the French and the German system, we systematically assess the dominance of the after-tax distribution over the pre-tax distribution in both countries separately. Obviously, no complete dominance of the after-tax distribution over the pre-tax distribution can be achieved, be it in Germany or in France. We therefore focus on the empirical cut-off lines. In the French case, OSOD would be verified provided that we would restrain the population to its 17 % fractile, which is equivalent, in terms of income level, to a household income below 11900 euros. Would we not bother of the singles, then OSOD would be verified provided that we would restrain the population to have an income below 20000 euros which is equivalent to the bottom 39 % of the population. And so on for each group (figure 3).

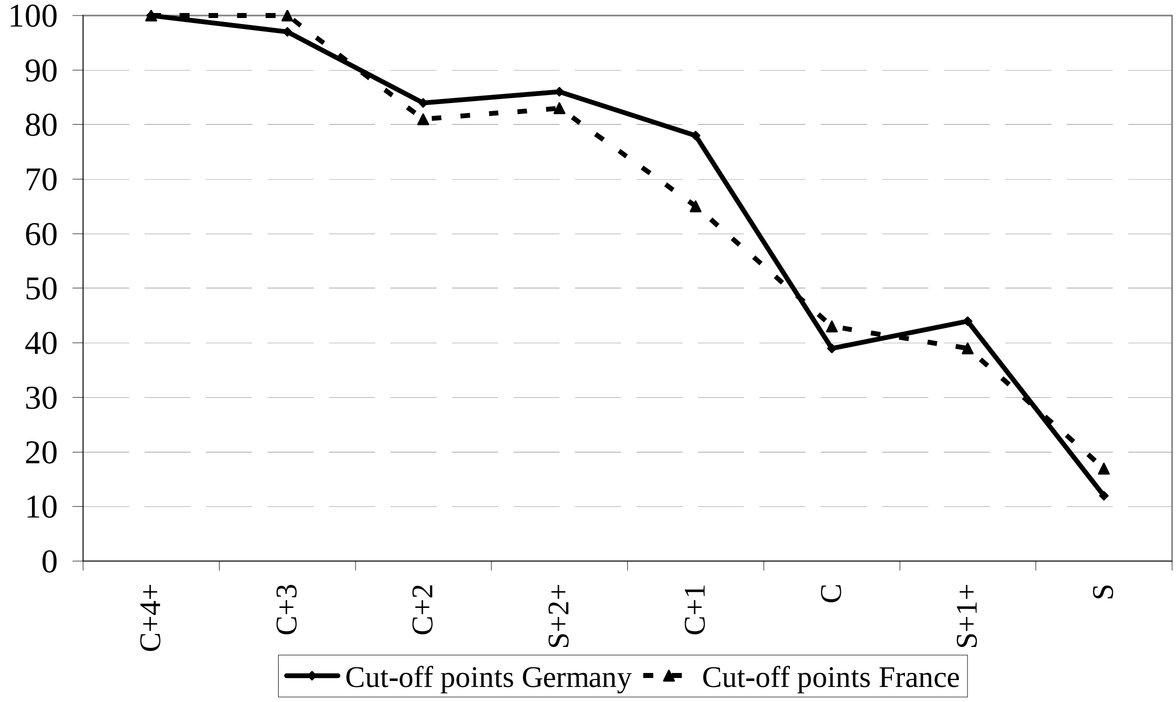


Figure 3: Empirical dominance cut-off line, Germany and France, 2001.
Sources: Calculations of the author GSOEP2002 and ERF2001.

3 The SAS macros

3.1 *Different SAS macros an hand*

The SAS macros are divided in four parts. Precise documentation of the functioning of each macro is to be found directly in the source code of the macros, which can be obtained under the following address mailto:-dg75-f350@insee.fr**.**

3.1.1 Overall Toolbox

``Overall Toolbox'' contains the following macros: **%creationCdf**, **%creationCdf2**, **%creationCdf3**, **%graphs**, **%graphs2**, **%graphs3**, **%graphRedistribution** which are needed by the macros of the other parts.

3.1.2 Lorenz Dominance

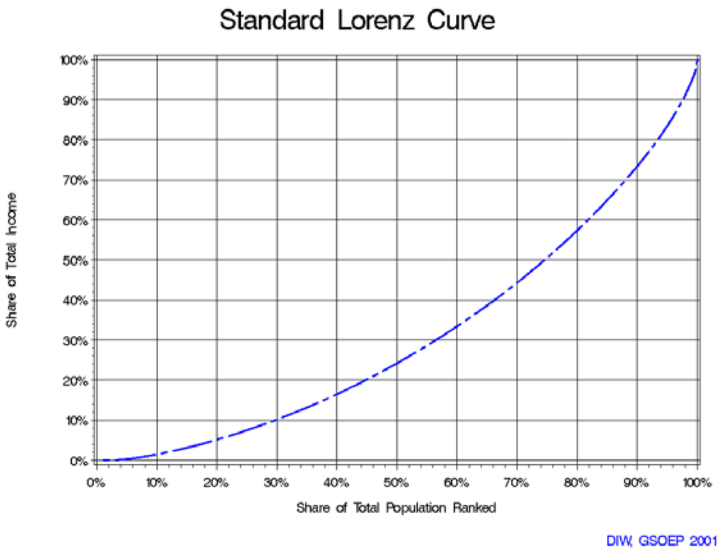
``Lorenz Dominance'' contains the following macros: **%SLorenz**, **%SLorenzCurve**, **%GLorenz**, **%GLorenzCurve**, **%SLorenz2**, **%GLorenz2** which compute the classical Lorenz curves, and compare them. Examples of output produced by these macros are given in Table 3 and 4.

3.1.3 Stochastic Dominance

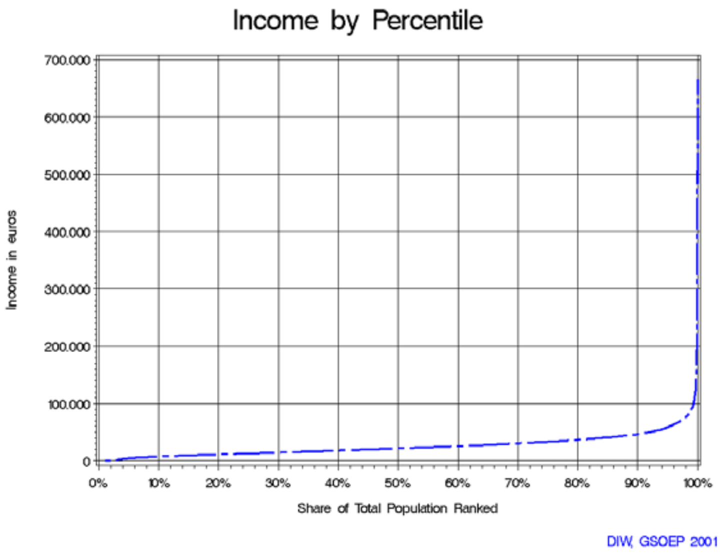
``Stochastic Dominances'' contains the macro **%dominance** which computes and graphs first and second order stochastic dominance. Examples of output produced by this macro are given in Table 5.

3.1.4 Sequential Stochastic Dominance

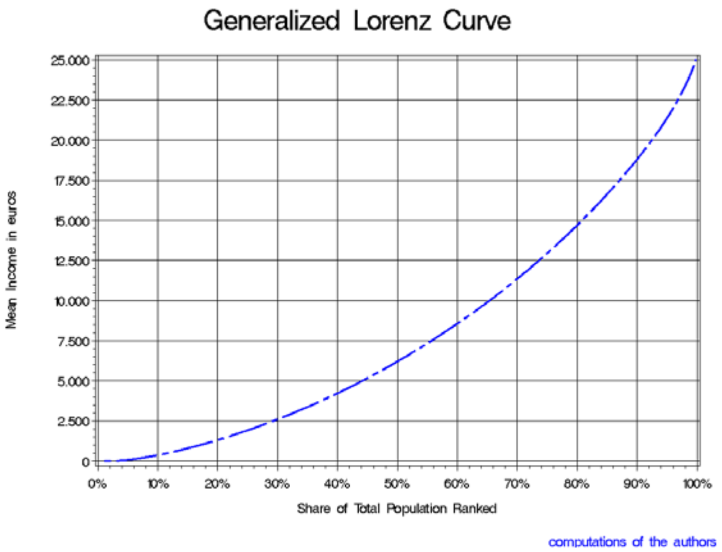
``Sequential Stochastic Dominances'' contains the following macros: **%prepareTable**, **%sequentialDominance**, **%creationSeqBefore**, **%creationSeqAfter**, **%final**, **%findRestr2nd**, **%interNeedsRedistribution**, which compute and graphes first and second order sequential stochastic dominances. Examples of output produced by these macros are given in Tables 6 and 7.



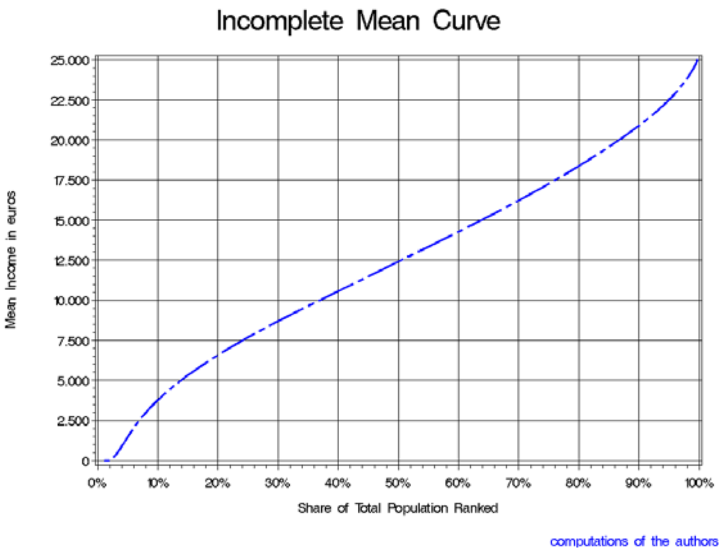
(a) Lorenz



(b) Income by percentile

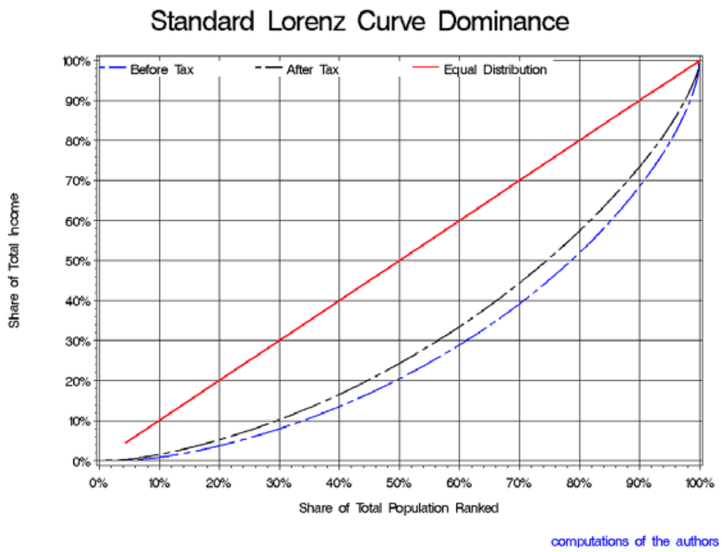


(c) Generalized Lorenz

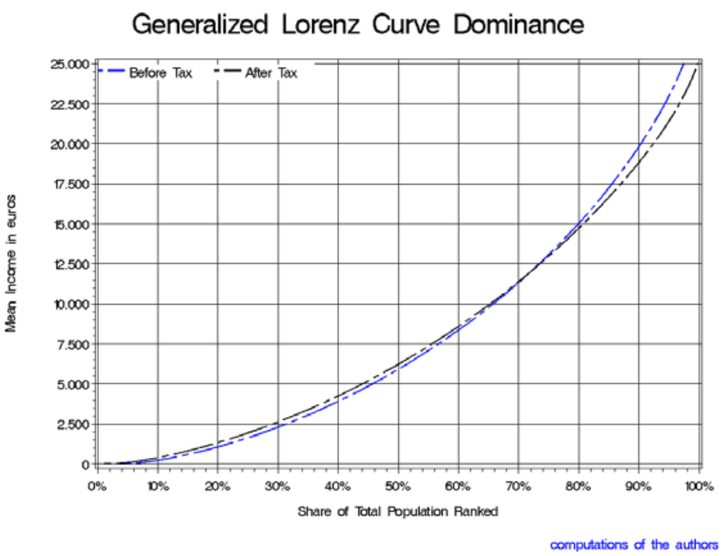


(d) IMC

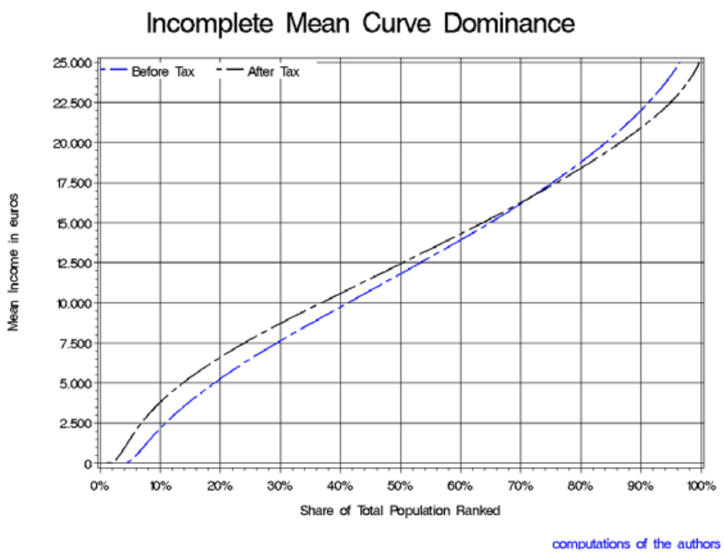
Figure 4: Lorenz Curves



(a) LD

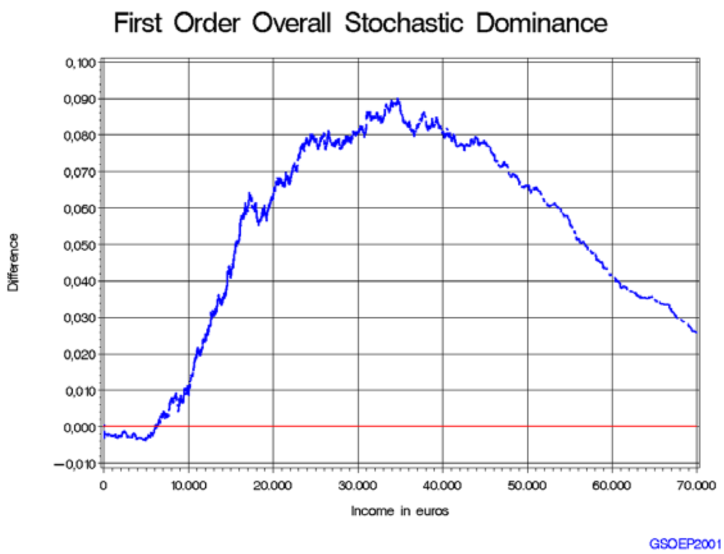


(b) GLD

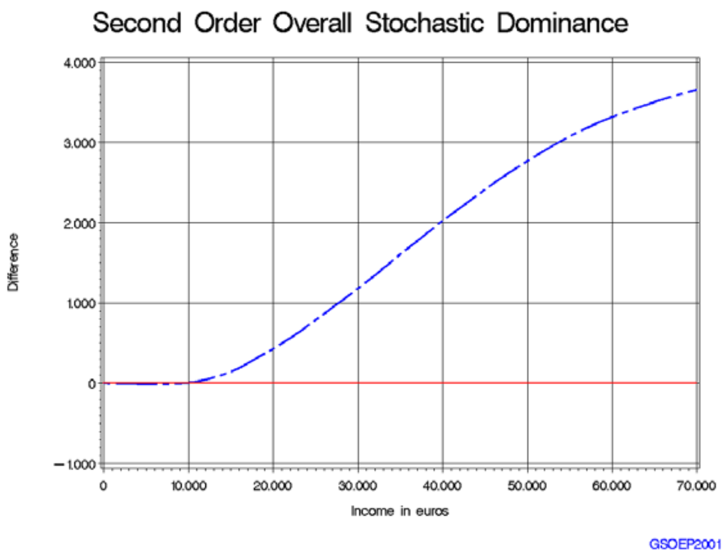


(c) IMC

Figure 5: Lorenz Dominances

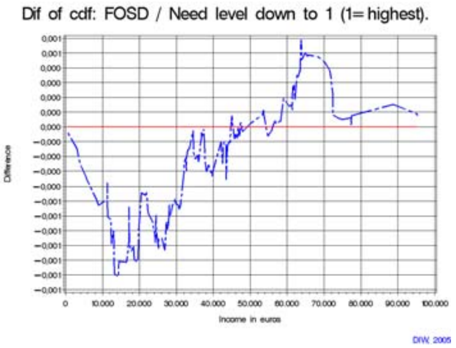


(a) FOD



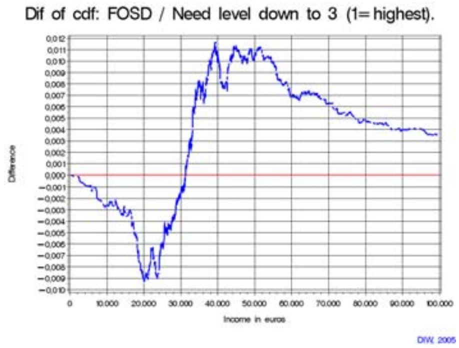
(b) SOD

Figure 6: Overall Stochastic Dominances

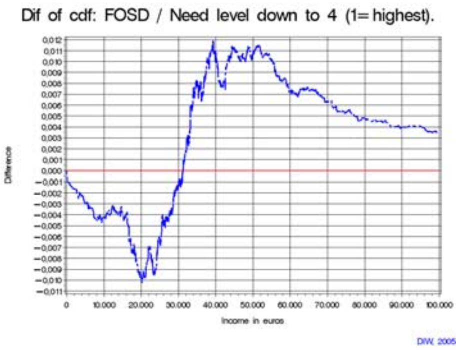


(a) Needs = 1

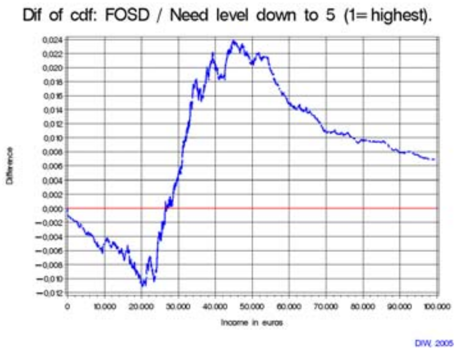
(b) Needs = 1-2



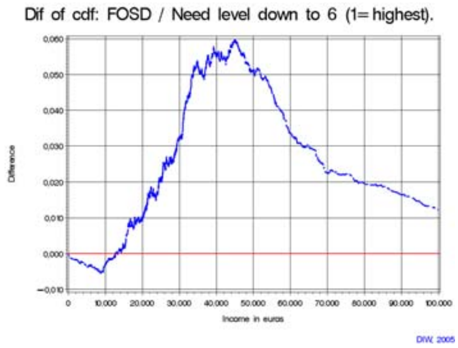
(c) Needs = 1-3



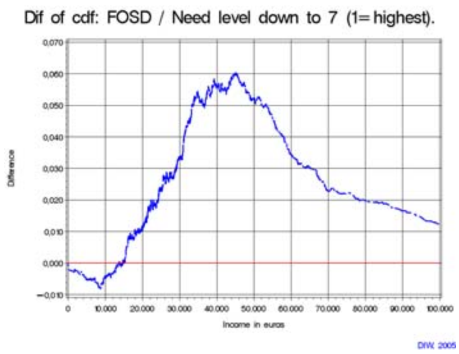
(d) Needs = 1-4



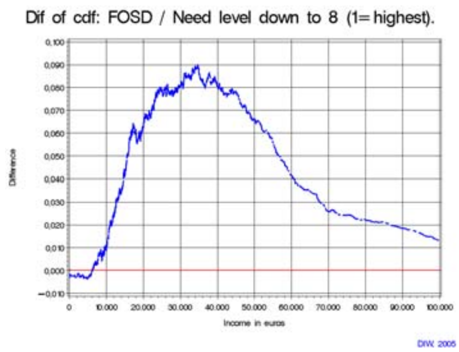
(e) Needs = 1-5



(f) Needs = 1-6

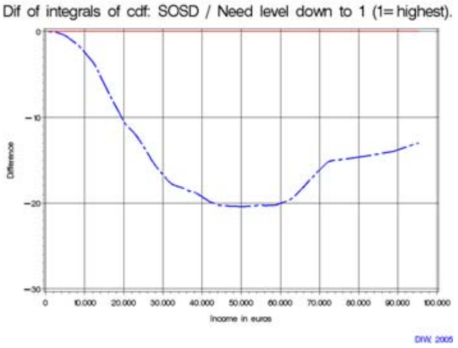


(g) Needs = 1-7

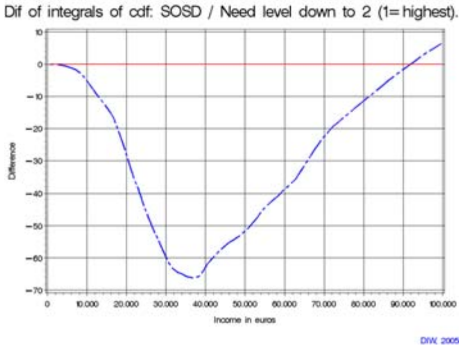


(h) Needs = 1-8

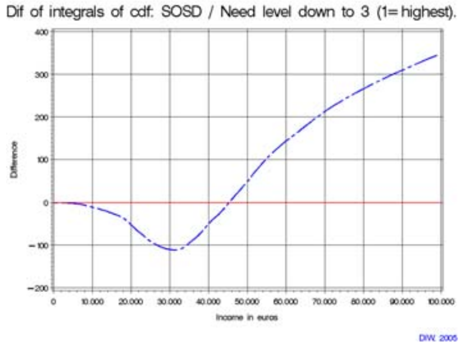
Figure 7: Sequential First Order Stochastic Dominance



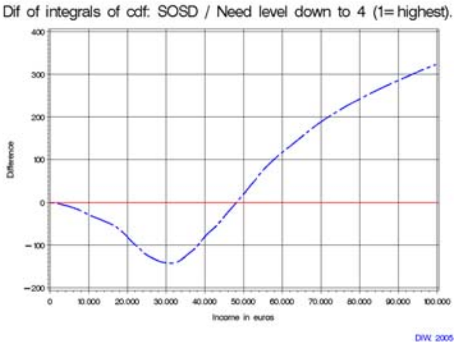
(a) Needs = 1



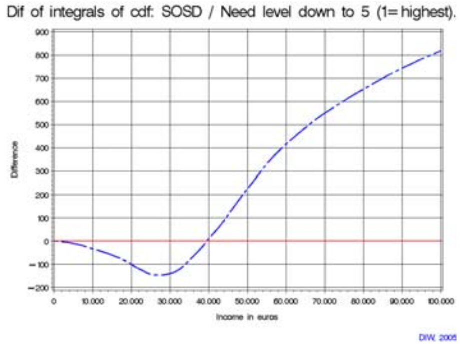
(b) Needs = 1-2



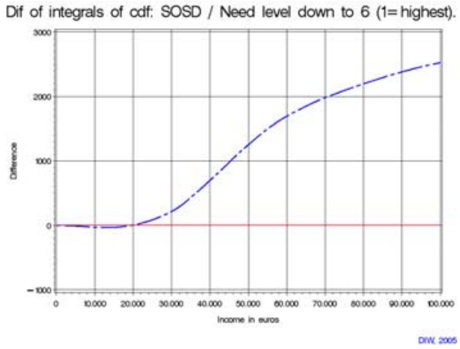
(c) Needs = 1-3



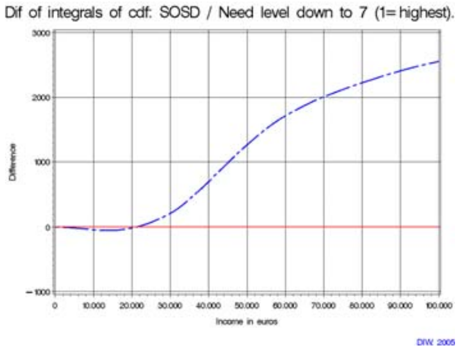
(d) Needs = 1-4



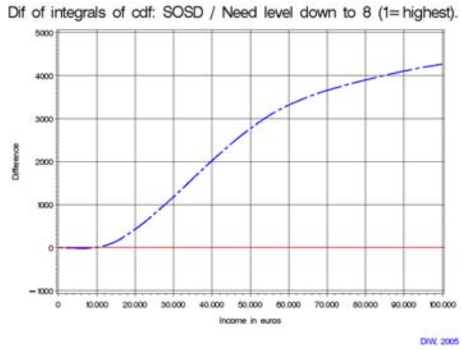
(e) Needs = 1-5



(f) Needs = 1-6



(g) Needs = 1-7



(h) Needs = 1-8. Overall second order dominance

Figure 8: Sequential Second Order Stochastic Dominance

3.2 *Example of application*

```
/* Use of Set 1 */
%SLorenz(table=germanyexample, library=work, variable=aftertax_hh, pond=hhrfk);
%SLorenz(table=germanyexample, variable=aftertax_hh, pond=hhrfk);
%SLorenzCurve(source="DIW, GSOEP 2001");
%GLorenz(table=germanyexample, variable=aftertax_hh, pond=hhrfk);
%GLorenzCurve (top=25000, step=2500);
%SLorenz2(table=germanyexample, variable1=pretax_hh,variable2=aftertax_hh,
pond=hhrfk, seuil=1, label1="Before Tax", label2="After Tax") ;

%GLorenz2(table=germanyexample,variable1=pretax_hh,variable2=aftertax_hh,
pond=hhrfk, seuil=1, label1="Before Tax", label2="After Tax", top=25000,
step=2500);


/* Use of Set 2 */
%dominance (variable1=pretax_hh , ponderation1=hhrfk , table1=germanyexample,
variable2=aftertax_hh , ponderation2=hhrfk, table2=germanyexample, librairie2=
work, pass_source="GSOEP2001", pass_top1 = 100000, pass_top2= 100000 );


%dominance (variable1=pretax_hh, ponderation1=hhrfk , table1=germanyexample,
variable2=aftertax_hh, pass_source="GSOEP2001", pass_top1 = 70000, pass_top2=
70000 );


/* Use of set 3 */
%prepareTable (table=germanyexample, librairy=work, work_librairy=work,
needs_classes=8, needs= needsshort);

%sequentialDominance(needs_classes=8,need_var=needsshort,
table=germanyexamplenomissing, library=work, variable1=pretax_hh,
variable2=aftertax_hh, pond=hhrfk);

%creation_seq_before(needs_classes=8, library=work, income=pretax_hh);
%creation_seq_after(needs_classes=8, library=work, income=aftertax_hh);
%final(needs_classes=8, library=work, income1=pretax_hh, income2=aftertax_hh);
%graphs2(needs_classes=8, library=work, top=100000, source="DIW, 2005");
%graphs3(needs_classes=8, library=work, top=10000, source="DIW, 2005");
%findRestr2nd (library=work, file="C:\WorkSpace\HBS\Stops.xls",
needs_classes=8,
library_perc_ref=work, data_perc_ref=germanyexample, var_perc_ref=pretax_hh,
weights_perc_ref=hhrfk, precision=1);

%interNeedsRedistribution(library=work,                    table=germanyexample,
variable1=pretax_hh,
variable2=aftertax_hh, weights=hhrfk, needs_classes=8, needs=needsshort);
```

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