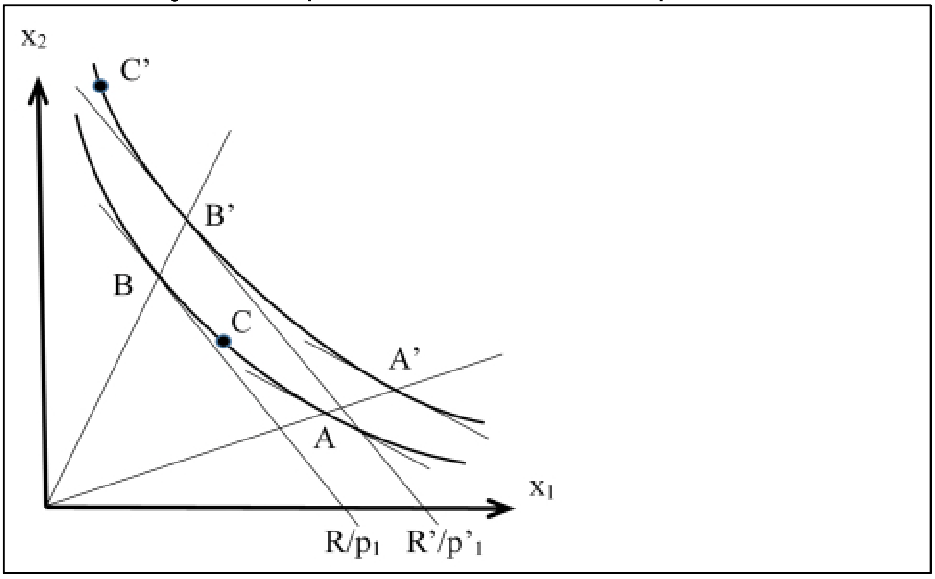
Online appendix C1 – How do volumes relate to consumer utility?

National accounts do not measure welfare, but the notions of volume or real income that they seek to measure are difficult to characterize without reference to the notion of consumer utility. In the end, it is the comparison of the services rendered by the various goods and therefore their respective utilities that constitute the natural yardstick for aggregating changes in their quantities. Their supposed link with marginal utilities is what justifies using prices for this aggregation. The concepts of volume, real income and utility therefore overlap, yet without being strictly equivalent. How do they relate to each other?

We can start by asking the question in the case of a representative consumer whose preferences are homothetic: preferences are said to be homothetic when the indifference between two baskets x and x' implies the indifference between baskets λx and $\lambda x'$, for any value of λ. The resulting shape for the indifference curves is shown in Figure C1-I. Homotheticity means that utility increases in the same way for a given relative increase in quantities, regardless of the direction in which one radiates from the origin point, i.e., on the graph, when moving from point A to point A' or from point B to point B'. A property that results from this is the parallelism of the tangents to these indifference curves when moving along a given radius.

Figure C1-I – **Equivalent income with homothetic preferences**



When preferences are of this form, they can be represented by a cardinal utility function $F(G(x_1, x_2))$ where G is homogeneous of degree 1 and F is any monotonically increasing transformation. Component G describes the agents' ordinal preferences, i.e. how they interclassify the different baskets of goods and the degree to which changes in the quantities of x_1 offset those of x_2. However, it ignores the fact that doubling total consumption does not necessarily lead to a doubling of utility or cardinal well-being. It is the function F that accounts for this phenomenon, if $F"(.) < 0$.

In such a case, it can be said that volume-price decompositions or the calculation of real income both amount to estimating the G component of the overall utility. National accountants do this using chained indexes. By chaining the effect of small changes in quantities with prices that are revised at each period, one avoids the problem illustrated in Figure I of the text, that of weighting by baseline prices that may progressively become less and less representative of the relative utilities of goods. Formally, this calculation of chain-linked volumes at the previous year's price can be seen as a discrete-time approximation to the continuous time Divisia formula (Hulten, 1973):

$$\exp\left(\int_0^T \frac{p_1(t)dx_1 + p_2(t)dx_2}{p_1(t)x_1(t) + p_2(t)x_2(t)}\right)$$

What justifies this aggregation based on prices is the assumption that they are proportional to the marginal utilities of the various goods, which will be the case if consumption is optimised at each period, i.e.

$$p_i(t) = \mu U'_i = \mu . F'(G). G'_i.$$

The integral is rewritten in this case:

$$\exp\left(\int_0^T \frac{G'_1(t)dx_1 + G'_2(t)dx_2}{G'_1(t)x_1(t) + G'_2(t)x_2(t)}\right)$$

It is further simplified by taking into account the degree 1 homogeneity of the G function, which makes it possible to apply Euler's identity. $G = G'_1.x_1 + G'_2.x_2$. It boils down to:

$$\exp\left(\int_0^T dG/G\right) = G(T)/G(0)$$

showing that the volume indicator mirrors the G function, up to an undetermined scale factor.
The same result holds if volumes are calculated by deflating income R with a constant utility price index (CUI). The CUI tells us by how much nominal income must be increased to maintain the same utility level U when moving from prices p to prices p'. With homothetic preferences, this CUI depends only on prices and not on the utility level U that we are trying to preserve. By definition of the CUI, the quantity $(R(T)/R(0))/(CUI(T)/CUI(0))$ will be constant if the combined variations in prices and income cause us to remain on the same indifference curve, and it will increase proportionally to x if we move along the radii along which the consumption and price structures are invariant. This again reproduces the properties of the function G, always up to a scale factor.

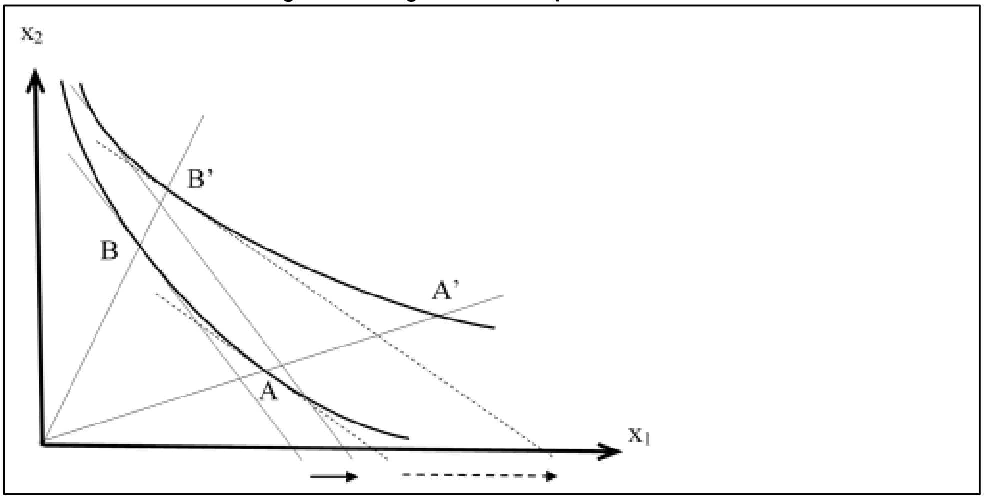
A third way of comparing any two baskets such as C and C' is the equivalent income method. It consists in setting a reference price system and comparing the indifference curves on which these two baskets are located by calculating the minimum incomes required to reach these curves, for this given price system. In figure C1-I, taking the relative price system corresponding to point B, this corresponds to the values R/p_1 and R'/p'_1 plotted on the horizontal axis, having taken good 1 as the *numéraire*. With homothetic preferences, this way of comparing the indifference curves is again independent of the price system that has been taken as a reference.

We thus have three methods providing the same result. The G function they allow to reconstruct cannot be equated with well-being because it ignores the fact that well-being does not necessarily increase in proportion to income or to the basket of goods that this income can buy. One way of pinpointing this difference is to avoid this term and rather speak of standard of living or of the purchasing power of income: having twice as much income at constant prices doubles the standard of living or the purchasing power without the final well-being necessarily changing in the same proportion. Real income or standard of living are nonetheless very much linked to the form of the U function: they are one of the possible scalarisations of the ordinal preferences underlying this U function.

What happens to this result when we abandon the hypothesis of homothetic preferences? Things become more complicated, of course, but in a way that, once again, can only be understood by reference to the concept of consumer utility. As far as chaining is concerned, the problem will be that its result becomes dependent on the path followed between dates 0 and T. For deflation by a CUI, the problem will be that this CUI depends on the level of utility that is taken as a reference. For the equivalent income approach, the problem will be that the result will depend on the price system that is chosen as a reference: this is what is illustrated in figure C1-II with a difference in the equivalent incomes associated with curves AB and A'B' which is not the same depending upon whether one takes as reference prices those associated with point A (dotted line arrow) or point B (bold line arrow).

If the origin of the problem is the same in all three cases, the consequences are not the same. There is no possible solution for chaining, which is one of the limitations of this method. In practice, what is done is only to forbid its use in case of excessively erratic temporal evolutions: the risk would be, for example, to conclude that there is a change in the volumes produced or consumed when a movement upwards and then downwards exactly returns to the initial conditions. For the dependence on U of the CUI or the dependence on p of equivalent income, they can be resolved by making explicit the levels of U or p that are taken as references. Generally speaking, it is both the notion of volume and utility that are necessarily relative: the same pair of relative variations in the quantities of two goods cannot represent the same increase in overall volume depending upon where one stands in the middle of the set of indifference curves.

Figure C1-II – **Equivalent income with arbitrary preferences: results change according to the set of prices used as a reference**



Online appendix C2 – Sustainability assessment in a capital-based approach

A capital-based sustainability indicator aggregates current changes in stocks of the different kinds of assets using imputed prices that reflect their true importance for future well-being. The theoretical principle is as follows. One notes $K_i(t)$ the stocks corresponding to date t, i ranging from 1 to m. The other variables describing the socio-economic and environmental system are n variables C_j describing both "consumption" and "actions" at time t, always understood in a broad sense. It is these C_j's which, in the spirit of the model in the first section of the article, are the main arguments of the current well-being function, but K_i's can also be some of these arguments hence a general specification

$$U(t) = U(C_1(t), \dots, C_n(t), K_1(t), \dots, K_m(t))$$

For given consumption behaviors, the dynamics of the system is described by a system of equations giving the evolution of the $K_i's$, i.e.:

$$dK_i/dt = R_k(C_1(t), \dots, C_n(t), K_1(t), \dots, K_m(t))$$

generalizing the equation $dK\,(t)/dt = f(K(t), L(t)) - C(t) - \delta K(t)$ we had in the basic model with one single all purpose good, allowing for as complex dependencies as one may wish. For instance, the capital stock K_i may decrease as a function of withdrawals made for all the consumption or actions C_j's, and either self-regenerate or depreciate more or less rapidly depending upon its current level (incorporating the possibility of threshold effects) and possibly on the current levels of other types of capital, if there are interactions between the dynamics of all these different assets.

On these grounds, we consider a "program", i.e. a long-term projection of all the variables of interest, on the basis of behavioral hypotheses that do not necessarily need to be optimal in any sense of the term. The projection of the sequence of future welfare levels can be summarized by a global discounted intertemporal utility index $W(t)$ defined as

$$W(t) = \int_{t'>t} e^{-\rho(t'-t)} U(t')dt',$$

which corresponds to the usual definition of wealth as the discounted flow of services that can be rendered by the initial stocks of resources $K_i(t)$, the services here being directly measured in terms of utility. Current well-being is said to be "sustained" along this trajectory if future values $U(t')$ never fall below $U(t)$. Current well-being is said to be "sustainable" if there is at least one path, possibly different from the path under consideration, where well-being never falls below this value.

A necessary condition for well-being to be sustained is to have $dW\,(t)/dt > 0$. Indeed,
$U(t') \geq U(t)$ for all $t' > t$ requires

$$W(t) \geq \int_{t'>t} e^{-\rho(t'-t)} U(t)dt' = U\,(t)/\rho,$$

hence

$$dW\,(t)/dt = \rho W(t) - U(t) \geq 0.$$

This $dW\,(t)/dt$ can then be rewritten as $\Sigma_i(\partial\, W/\partial\, Ki)\; dKi/dt$ which is interpreted as a generalized net savings rate valuating the different types of assets according to their marginal contributions to the wealth indicator $W(t)$. For current well-being to be sustained, this net savings rate must be greater than or equal to zero, this condition being necessary but not sufficient.

A positive or zero net savings rate is also a necessary condition for current well-being to be sustainable (rather than "sustained"), but only if the growth path is optimal for the utilitarian objective $W(t)$ (Hamilton & Clemens, 1999). Again, this is a necessary and not a sufficient condition. Even under the optimality hypothesis, positive adjusted net savings does not guarantee sustainability (Pezzey, 2004; Asheim, 2007). Conversely, the positivity condition is not necessarily necessary if the reference trajectory is not an optimal trajectory in the sense of maximizing $W(t)$: current well-being may not be sustained along such a trajectory, for example an extremely myopic trajectory of massive overexploitation of resources, without ruling out that it may be sustained along a more farsighted alternative trajectory.

The message to be drawn from the index therefore varies according to the context. One of its interesting features is to allow overriding the opposition between strong and weak sustainability: if one of the assets becomes crucial for the evolution of future well-being at a given date, for example when approaching a critical threshold on its spontaneous renewal, this should result in an infinitely large imputed price relative to that of the other assets, preventing any form of compensation for its decline by increases in other capital goods. One can for instance retrieve what the ecological footprint postulates *a priori* by giving a zero implicit value to all assets other than renewable natural resources. At the limit, one may even be led to assign a negative value to one or more K_i's if these have negative externalities on the regeneration of other assets more fundamental to future well-being. Simulations illustrating this type of property can be found in Fleurbaey & Blanchet (2013).

This being said, what this formalism shows is, above all, the extent to which the exercise differs from a classical statistical exercise. First of all, it shows that one cannot be satisfied with valuing environmental damage at the cost of restoring the environment. It is not the cost of restoration that spontaneously indicates how a degradation impacts future well-being, except under an assumption of optimality where the marginal cost of restoration and marginal damages would exactly balance each other. More generally, a calculation of adjusted net savings based on market prices would only make sense if the market prices of all assets were true indicators of their marginal contributions to future well-being. However, the inefficiency or non-existence of a single market is theoretically sufficient to distort price signals across all markets. The solution to the problem cannot therefore be reduced to combining market prices for assets that have one such price and imputed prices for those that do not: all prices must be re-imputed on the basis of complete economic and environmental projections.

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