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Domestic Product in Volumes, Value Added Double-Deflated, and National Income in Real Terms, Do They Fit Together? An Analysis of Concepts

1. Introduction

If ever national accounts came under religious scrutiny of a fundamentalist regime, they might give appearance of cultural bias. For is not the concept of Trinity firmly embedded in them, a concept defended by only one religion in the world, and that religion, in particular, within which the national accounts came into being? And is the mystification of that trinity not carried out to a point where it may safely be called „Holy"? The trinity of domestic product, value added and national income is the basis of all teaching in national accounts, a teaching that begins by stating that the three concepts are equal, in principle, and then proceeds by explaining, how they are not, a true mystery, indeed.

Fortunately, presentation of the accounts under a confessed laic government shields us from such imagined dangers[1], and the problem of trinity appears in a more technical disguise. Given that the concepts of domestic product, value added and national income have been clarified to a degree that the passage from one to the other is well-defined, at least statistically, albeit not philosophically, in nominal terms, the same does not hold when corrections for changes in prices are applied. The procedure is unnecessary and even meaningless, as long as one year and one country are considered only, but they are so much more required when it comes to comparisons over time and space. Domestic product is then compiled in volumes, value added is being double deflated, and national income changes from nominal to real income. How does the trinity-in-unity paradigm behave under these more severe conditions? Is unity still attainable, or, on the contrary, do the deflated variables naturally diverge, due to their different deflating procedures? Are the new balances created in this context meaningful statistical results, or just an expression of lacking coherency in compilation? The theory of national accounts is incomplete as long as it roams within the realm of nominal values alone, not addressing this last stage of compilation where all accounting ends up necessarily, before its results appear in a form that can be fully used.

In this paper, an attempt is made to establish coherence of national accounts after deflating, on the assumption that such coherence is desirable. The opposite assumption that it is not, but, on the contrary, that coherence in real terms is meaningless, may also be maintained, of course, but for the purpose of brevity we leave it aside here. Our analysis then proceeds as follows. First we recall the basic laws assuring coherence between domestic product, value added, and national income in nominal terms. Second, we will re-iterate the present state of the art of deflation as represented by the UN System of National Accounts (SNA), followed by a value-theoretic assessment of these recommendations, and, third, an ensuing innovative proposal for establishing coherence of the three fundamental concepts of national accounts after correction for moving prices. Application of the idea to an existing system of national accounts is intended, but not included in this paper. Instead, a simplified model of an economy will be invented for illustrating the proposal.

[1] Assuming us people to be as laic as our government, Webster's description of trinity might as well be reproduced fvor information: „The union of three persons or hypostases (the Father, the Son, and the Wholy Ghost) in one Godhead, so that all three are one God as to substance, but three persons or hypostases as to individuality." (Webster's New College Dictionary, Springfield, Mass. 1953, p. 909). German language has managed to put this sentence in one word: „Dreieinigkeit". The infȍuence of this ideology on th author will become visible in the paper.

2. Coherence in nominal terms

Income stems from production in a market economy. Following this basic axiom of income theory national accounts begin be collecting the value of all output of goods and services generated in the economy. The resulting product account juxtaposes supply and use of the products, and may be set up for each product individually. Turning from the product to the process behind, the production account assembles the products of each producing unit, juxtaposing them to the input of products used up in the production process. The total value of the latter being necessarily smaller than the value of output in a market economy, the balance is given the meaning of the value added to inputs through the process. Value added can be compiled for each producing unit, usually an establishment. Again by definition, value added determines what income may be paid out to the participants in the process as a means to acquire the created products. This is shown on the income accounts following the different stages of the distibution process. Coherence is thus assured by statistical definition,

(1) domestic product = value added = income.

This, however, is only one half of the matter. The other half is represented by the question of why three different names are given to one and the same number. If domestic product equals value added and this again equals national income, in principle, why not use one word only, and call all three „product“, for example? Are the three words synonyms? If not, is there a difference in meaning attached to these three terms, in spite of their statistical identity? Do the three concepts differ in quality, even if they are equal in quantity, by definition?

Rather than venturing into the semantic background and theoretical history embedded in these terms we turn directly to the accounts through which they are represented. Following a useful practice introduced by the SNA we set up the accounts of a model economy, simplified in an appropriate way, so as to answer the question of meaning. Table 1 shows the model. It consists of two parts, one called production account, the other income account. This corresponds to the main division of a market economy in two complete, and yet intertwined circuits of flows, supply and use of goods and services, on the one hand, and receiving and expenditure of income, on the other, or, to put it more economically, the distinction between division of labor in transforming natural resources, and individual property rights in determining access to the results of this labor.

Table 1: Accounts of a model economy in nominal values for year 0

a) Production accounts (billion Euros)

	Prod. 1	Prod. 2	Cons.	Cap. fo.	Export	Import	GDP	Output
Product 1	100	90	80	70	60	40	170	360
Product 2	60	50	40	30	20	10	80	190
Value added	200	50						
Totals	360	190	120	100	80	50	250	550

b) Income accounts

Income generation
(Billion Euros)

Compensation of employees paid	120	Value added	250
Taxes on production and imports	50		
Operating surplus	80		

Income distribution

Property income paid	90	Operating surplus	80
		Taxes on production and imports	50
		Compensation of employees received	116
		Property income received	104
National income	260		

Several simplifications have been made for the purpose of the analysis without invalidating its findings, hopefully. As has been evident all along we do not differentiate between gross and net, we flatly ignore this attribute. Consumption of fixed capital is an item that may or may not be carried on through the accounts, and we may safely leaving it aside as an optional way of expressing anyone of the variables in question. Another simplification concerns valuation. Valuation of products and derived variables is a complex problem, being signified by a diversity of concepts from „market prices“ through „producers' prices“ to „basic prices“ and“ factor cost“. Under a value theoretic perspective we must ask which of the four is preferable if we want to simplify. Given the fact that the national accounts are based on the transactions principle, the value of a product is determined by the amount of liquid assets to be mobilised in order to pay for acquiring the product in question. The view is corroborated by the requirement of coherence with financial accounts which form the inevitable complement to the real accounts in the overall system. Hence we decide for valuation at market prices throughout our simplified system. Finally, we need not present the system of accounts in full, but only those accounts that show and determine the three concepts under consideration, namely domestic product, value added and national income, the rest following logically from them.

Looking at the economy in terms of the goods and services it creates, domestic product is defined as the value of those goods and services that are produced within a territory within a year and not used up within the same territory, the same year, but outside these limits, which is called „final use“[2]. This yields the standard formula:

(2) domestic product = final consumption + capital formation + exports - imports,

and is shown in the second quadrant of table 1a. Domestic product at market prices amounts to 250 out of a total output of goods and services of 550. The the third quadrant of table 1a

[2] It may be remembered that this satistical definition does not coincide with the economic definition acsording to which only consumption is the final purpose of production.

shows the production account in the strict sense. For each process output is balanced against intermediate inputs yielding value added of 200 and 50. While the arithmetic of the table is self-explanatory, its interpretation is not so. Is the value created in the production process being better measured including taxes on products and excluding subsidies, or better not? The SNA recommends the second alternative, which may be justified by pointing out that the producer does not see this part of his receivables but passes it on to the government, as value added tax, for example. On the other hand, if market value of the output is the realized value, as demonstrated by the fact that it leaves the business accounts of the producer to enter into those of the consumer at that size, that value can hardly disappear into some third pocket, before it has been properly added to the product. The issue will turn up again when we deal with double deflation.

Table 1b sketches the income accounts by showing the bridge between value added and national income. Following a terminology introduced in chapter XVI of the SNA by way of treating the terms of trade effect, we may say that value added equals domestic income, which turns into national income by incorporating primary income transactions with the rest of the world. Still, as this treatment determines only the qualification of domestic vs. national the question remains in which way value added (domestically) is different from (domestic) income qualitatively, if by definition they are always equal in size[3]. A brief answer may run like this: Value added measures the productive effort exerted in establishments, while income comprise the legal claims arising to the product of the activity. The two are not identical on an individual basis, but for the economy as a whole there cannot be more, nor less claims than products or activity. This issue will also be brought up later, in the context of deflation.
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3. Deflated aggregates in the SNA

In presenting its recommendations on how to deflate nominal variables the SNA follows a long standing tradition carried on through four revisions. It works out all accounts purely in nominal terms. Only in chapter XVI it turns to problems associated with nominal values, and discusses price and volume measures. It puts thus the problem of real values at the periphery of the system, similar in importance to population (ch. XVII), classifications (ch. XVIII), use of national accounts (ch. XIX), and finally social accounting matrices (ch. XX), and satellite accounts (ch. XXI). In the „Reader's Guide" dividing th chapters into five groups, beginning with introduction and overview (group I), basic tools (group II), the chapters of group III present the central framework, ranging from chapter VI to chapter XV. The border to the perophery runs right before chapter XVI.

The border-line treatment of techniques of deflation is natural, in that it corresponds to tradition as it evolved in the development of national accounts. Coherency of the system must be sought in nominal values, first of all, and it is difficult enough to achieve as witnessed by the preceding 15 chapters. Yet, from the point of view of the user of national acounts priorities are different. Even the most elementary system of national accounts is useless if it does not provide for intertemporal and international comparison. The SNA itself acknowledges that fact, if only implicitely. When writing:

„The SNA consists of a coherent, consistent set of macroeconomic accounts and tables designed for a variety of analytical and policy purposes. Nevertheless, certain key aggregates of the System, such as GDP and GDP per head of population, have acquired an identity of

[3] The question is treated extensively in Reich (2001).

their own and are widely used by analysts, politicians, the press, the business community and the public at large as summary global indicators of economic activity and welfare.“ (SNA para. 1.68)

the importance of proper deflation procedures, without which all nominal accounting coherency and detail would be fruitless, clearly comes to mind. Comparison in time and space forms the basis of many an economic analyses, and to arrive at some measures in real terms is thus an essential step in the elaboration of the system without which it can not claim to be complete. “L’établissement de séries en volume, c’est-à-dire abstraction faite des évolutions nominales, est au coeur de la problématique des comptes nationaux.“ (Berthier 1999, p.).

In this spirit we read Chapter XVI of the SNA as a cursory introduction to the topic, summarising the present state of the art in the field, but we will not expect that it has achieved the integration of price statistics and national accounts required in order to build a common system of concepts, within which the shrine for the sacred growth rate of GDP may safely be embedded.

Chapter XVI is entitled „Price and Volume Measures“, and the greater part of the chapter deals with these two concepts, indeed. Only in the last section K a third term is introduced, called „real income“ and described as follows:

> „It is possible to deflate any income flow in the accounts, and even a balancing item such as saving, by a price index in order to measure the purchasing power of the item in question over a designated numeraire set of goods and services. By comparing the deflated value of the income with the actual value of the income in the base year, it is possible to determine by how much the real purchasing power of income has increased or decreased. Income deflated in this way is generally described as ‘real income’“. (SNA para.16.148)[4]

The operation of deflating nominal values is here not only applied to product transactions as elaborated in the main part of sections A- J, but carried over to income transactions in section K. And in doing so a new theoretical concept of value has been evoked, called „purchasing power“. We will build on this concept later. It represents an important variable in the national accouts the theoretical substance of which needs to be brought to light.

There is a serious problem in applying the concept of real income to national accounts, in the eyes of the SNA: Real incomes depend on the choice of a numeraire. As there may often be no obvious, or uncontroversial choice of numeraire there has alway been some reluctance to show real income in the national accounts. There is a large but inconclusive literture on this topic, but one point on which there is general agreement is that „the choice of P can sometimes make a substantual difference to the results... and this has prevented a consensus being reached on this issue.“ (SNA para. 16.153) On the other hand, it is highly desirable and for some countries „vitally important“ to calculate real income, so the SNA delegates the choice of appropriate deflator to the statistical authorities in a country, finally, „taking account of the particular circumstances of that country.“ (SNA para 16.153) In summary, finding a numeraire for the measurement of real income is highly wanted, but the SNA does not have one.

[4] Vanoli deplores the use of the adjective in the term ”real income”, giving preference to “income in reel terms”. But he does not give a reason, or explain the difference. In our view both terms are sysnonyms, and have the same meaning, but the longer one sounds more technical and impressive.

A particular point demanding SNA attention is the treatment of international trading gains which bridges gross domestic product at constant prices and real gross domestic income. Here consensus is not difficult and the supplied formula well known:

$$T = \frac{X - M}{P} - \left(\frac{X}{P_x} - \frac{M}{P_m} \right). \tag{3}$$

where X are exports, M are imports at current prices, P_x and P_m are the corresponding price indexes, and P „a price index based on some selected numeraire".

The third and last part of the section on real income finishes off quite technically[5]. Assuming that measures of trading gains and trading losses are available, various real income aggregates are presented as part of the system:

		gross domestic product at constant prices
	+	trading gain or loss
	=	real gross domestic income
	+	real primary income receivable from abroad
	-	real primary incomes payable abroad
(4)	=	real gross national income
	+	real current transfers receivable from abroad
	-	real current transfers payable abroad
	=	real gross national disposable income
	-	consumption of fixed capital at constant prices
	=	real net national disposable income

Here the choice of the numeraire causes less headache. It is recommended that the purchasing power of these flows should be expressed in terms of a „broadly based numeraire, namely the set of goods and services that make up gross domestic final expenditure". The immediate question coming to mind at this point is why the same numeraire may not be applied to the trading gains, discussed before. But we get back to that. We leave the SNA at this point in order to make some specific comments on the text.

We have already noted the appearance of a third notion of deflated value „purchasing power" besides price and volume. The concept is rarely used in national accounts, mostly when it comes to international comparisons. Our suggestion will be to apply the concept to national aggregates as well. The problem of the choice of an appropriate numeraire is not as insurmountable as it seems. Actually it exists already and is being generally accepted, only that its acts under a different disguise, called „rate of inflation", as we will show.

Critical is the lack of coherence of the variables defined in chapter XVI. It seems that product and income hold together only at the upper most level of aggregation, as indicated by the title of the subsection: „Measures of real income for the total economy". What happens below the total is not clear. How „real gross national income" divides into compensation of employees and property income, how real net national disposable income divides betwen the sectors and

[5] The difference in style compared to the former parts of the chapter is noticable, and lends itself to guessing a shift in authors, the first being Peter Hill, the other André Vanoli. For a full account of the discussion see A. Vanoli (2002), p. 472 ff. Its result: "On a donc beaucoup discuté des principes, et le débat n'est certainement pas clos" (p. 475) may perhaps be read as an encouragment to writing this paper.

so on, is not explained, except for some cursory deliberations at the beginning of the chapter (para. 16.2). Coherency of deflated items must be made as explicit as it is for nominal items if answering to the claim of an integrated system is intended. Chapter XVI is far from doing that.

4. A theoretical move: tripartition of the concept of value

In our view techniques of deflation belong in chapter III of the SNA, because they are one of the tools needed to construct useful national accounts. Discussing flows and their aggregates in purely nominal terms is insufficient for arriving even at the most prominent economic indictors, growth rate of GDP and national difference in GDP per capita, let alone all the other aggregates, consumption, capital formation etc, which also underly the general bias incorporated in nominal values.

In order to give more power to this plea consider the following: Before entering a single number into any statistical table one must put the following information in its head: content, country, period, and last not least the unit of account. The first three are not a problem, the the fourth is one, oddly enough, although it is self understood. In national acounts the unit is evrywhere and at all times the national currency. Being so evident would be a reason not to bother about it, but in contrast to all other units used in statistics, the currency unit is not a physical one, but created by the economy itself. As a consequence, the meaning is not fixed universally and once for all times, but it varies with the object it is to measure. The discovery of the fact goes back to David Ricardo who was also tormented by it more than onyone else, because he clearly realised the consequence[6]. If the unit of account is not invariable, not "absolute", as Ricardo called it, but moves in time and between countries, how can you be sure that a change of a figure, measured in this unit is due to the oberved variable, and not to a variation of the unit itself? If the price of an apple rises, is this revaluation of the apple, or devaluation of the currency unit?

Later economists up until today have solved the logical circle by cutting it short. In the foot-path of what then became to be called microeconomic or neo-classical value theory, one refutes an answer by saying that the two effects are the same, the resaon being that in this theory, prices are only relative magnitudes. The price of an apple is it value relative to a pear or to any other commodity, all being equally admissible as measzrment units, since in relative terms they all can be transformed into each other. But in fact, in economic practice the question has been answered. An apples price rise is a devaluation of the currency, it enters directly into the rate of inflation. Central banks take the average price rise in an economy as the indicator of devaluation of the national currency.

While this is generally acknowledged in economic practice it also has consequences for the theory of economic value. It is perhaps not uncontroversial which numeraire to use for measuring the value of a currency, but a decsion has been taken, and this is incorparated in every price measurement. Price statistics do not measure prices of apples in terms of pears, and the pears also in terms of other goods ad libitum, as that would give a different price movement in each case. Prices are measured in the unit in which they are expresssed, namely the national curency.

This theoretical observation has repercussions on price statistics. The elementary time series

[6] See D. Ricardo (1952).

of prices $p_i(t)$ of a commodity i incorporates two different movements, the price change of the specific commodity in question, and simultaneously the change of the unit in which prices are expressed. If p_i is the nominal price, the real price π_i, meaning the price of the commodity relative to all other commodities in the sense price is usually understood in economic theory as an opportnity cost of exchange, is the nominal price corrected for the movement of the currency unit itself,

(5) $$\pi_i = \frac{p_i}{\lambda}$$

where λ is the so-called general price level, which measures in practice not prices of products but value of money. If the price series for apples shows a change from 100 to 105 between 1990 and 1991, and then a change from 120 to 125 between years 2000 and 2001, the changes of 5 units in each case are not comparable directly. For 5 units in the second case are less then in the first one. Comparison can be achieved indirectly by correcting for the devaluation of the currency that has taken plac in between the two periods. The statistician, in the price department as well as in the national accounts, will realise that their unit of measurement is not invariable in time, but follows a movement of its own, in contrast to measurement units used in other statistical departments. Money is not only a means of payment, a financial asset under custody of a central bank, but it is also the single unit of measurement for the real economy, and thus of paramount concern for the national accounts. This is why its treatment belongs in chapter III of the SNA.

The proper concept under which this phenomenon falls is purchasing power. As said above the concept appears in the SNA as a rather minor item. It deserves to be developed to its full size, because it is only then that it will show its beauty. Again it is the microeconomic tradition standing in the way here. When the SNA lingers about the indeterminacy of a numeraire, as reported above, it holds a subjective view of value, attached to indiviuals and founded in individual marginal utilities. These utilities being unobservable in themselves, are measured by comparing them against each other as marginal rates, revealed in relative prices as mentioned above. Purchasing power is the power of an individual to exchange his income for commodities. In this view it is impossible to arive at a general set of goods and service to function as a numeraire for money, because every individual has their own, which is not even constant over time.

Monetary politics has never cared much about such individual subtleties. Working on the assumption that individual economic welfare is served best by a stable currency, it has not hesitated to define a set of goods and services as the general numeraire serving as a measure of the purchasing power carried by the currency unit and tranported from one economic unit to the other in its function as a means of payment, having the same value by whom so ever it is being used in one economy at one point of time. There are two such commodity baskets in use at present, private consumption and GDP. In international comparisons, in fact, one has since long given up the idea of expressing indivual welfare by means of national statistics, but compares the purchasing power of currencies quite naturally. It should not take much to apply the same common sense to the change of a currency's purchaing power within its own economy, the only difference being that in international comparison the differences are discrete, while over time they occur continuously. Since in the microeconomic tradition „purchasing power" is a power attributed to a person or an economic unit, extending over its income, the point of view held in the SNA, it is advised to step aside and employ the more technical, albeit clumsy expression of purchasing power parity of a currency. There are people

who find the concept of a purchasing power of money „abstruse“, for reasons sketched above.

If national accounts and price statistics accept the concept of purchasing power parity as measure of variability of their unit of account, the attribute „real“ can be applied not only to income transactions, but to product transactions as well, because it is the same money that pays for both and if its value is corrected for inflation in one area of expenditure, it should so in the other[7]. And the use for this purpose implies a choice of numeraire. As GDP is the measure of overall production of goods and services of an economy, it is most natural to compare the value of its currency against this basket, and thus find the so called GDP-deflator as the standard for deflating all accounts of the system uniformly. In this way the accounts change from nominal to real values, and are then fully comparable as time series within themselves.

Choosing GDP as the numeraire implies thr rule that the volume of GDP equals its real value. Consequently, this does not hold for all other elementary product transactions or aggregates of the accounts. Here the volume change will be different from the real value change, the balance being held by the change of relative or real price. In a formula we have

(6) $$w_i = p_i\, q_i,$$

the usual bipartiton of a nominal value w_i in product class i into its two component price p_i and volume q_i. Separating the currency variation λ out of price p_i, as defined in equation 5, we arrive at

(7) $$w_i = \lambda\, \pi_i\, q_i,$$

a partition of value into three parts, each meaningfull for itself. We define the real value of a product transaction by

(8) $$\omega_i = \frac{w_i}{\lambda} = \pi_i\, q_i\,,$$

which is different from volume in that it incorporates the relative price of the commodity i.

All of these variables are functions of time, which may be assumed as continuous, as long as the economy is in equilibrium, which again is a condition that prices are in equilibium and can thus be measured throughout the economy („the law of one price“, required in applying the concept of market). Continuous change allows simple representation using of differential algebra, so that we can describe the three changes that happen to a nominal value by

(9) $$dw_i = \lambda\, d\omega_i + \omega_i\, d\lambda = \lambda\, \pi_i\, dq_i + \lambda\, q_i\, d\pi_i + \pi_i\, q_i\, d\lambda\,.$$

Thus a nominal value change consists of three components, the volume change as usual, and the real price change of the commodity, and, in addition, the change of purchasing power parity of the measurement unit. In the next and last section we show how this tripartition value can be applied to the model economy of table 1, in order to establish coherence of deflated national accounts in concept

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5. Applying the proposal to a model economy

As said in the beginning details that may well be essential in the further development of national accounts have been abstracted from in this paper, in order to address the specific problem of coherence, conceptually. As a useful tool for the purpose a model economy has been introduced , which will now be subjected to the suggested accounting methods. Table 2

[7] This point is argued more fully in Neubauer (1978).

shows the individudual steps taken in deflating nominal accounts.

Table 2:Product Accounts of a model economy for years 0 and 1

a) Nominal values year 0 (Currency units and prices of year 0)

	Prod. 1	Prod. 2	Cons.	Cap. fo.	Export	Import	GDP	Output
Product 1	100	90	80	70	60	40	170	360
Product 2	60	50	40	30	20	10	80	190
Value added	200	50						
Totals	360	190	120	100	80	50	250	550

b) Nominal values year 1 (Currency units and relative prices of year 1)

	Prod. 1	Prod. 2	Cons.	Cap. fo.	Export	Import	GDP	Output	Prices
Product 1	121	88	99	66	77	33	209	418	110
Product 2	66	66	42	42	18	18	84	216	120
Value added	231	62							
Totals	418	216	141	108	95	51	293	634	

c) Volumes year 1 (Currency units and relative prices of year 0)

	Prod. 1	Prod. 2	Cons.	Cap. fo.	Export	Import	GDP	Output		
Product 1	110	80	90	60	70	30	190	380	gdp deflator	
Product 2	55	55	35	35	15	15	70	180	Laspey	Paasche
Value added	215	45							113,2	112,7
Totals	380	180	125	95	85	45	260	560		

d) Real values year 1 (Currency units of year 0, relative prices of year 1)

	Prod. 1	Prod. 2	Cons.	Cap. fo.	Export	Import	GDP	Output
Product 1	107,4	78,1	87,8	58,6	68,3	29,3	185,5	370,9
Product 2	58,6	58,6	37,3	37,3	16,0	16,0	74,5	191,7
Value added	205,0	55,0						
Totals	370,9	191,7	125,1	95,8	84,3	45,3	260,0	562,6

Table 2a is a direct reproduction of table 1 showing a complete set of nominal values for year 0.[8] As explained above, valuation is at market prices throughout, which means that production taxes are included in GDP and value added. Table 2b adds a second year 1 to the first year, so that changes can be observed.[9] Assume that together with the nominal values of year 1 a set of price indexes is supplied by the departmnent of price statistics, indicating a price rise of 10 percent for product 1 and of 20 percent for product B. Dividing the price indexes into the

[8] In practice, nominal values are sometimes derived from volumes and prices, so the order of compilation is reversed, but in general, nominal values are given first and then deflated. We ignore the exceptions here.

[9] The observation that the separation of volume and price is never possible in a stationary state of the economy, but only when it is moving, seems trivial but is worth being remembered in this context.

nominal values of tables 2b yields the volumes of table 2c. It says that GDP of year 1 is 260 if valued at prices, and purchasing power parity, as we add, of year 0. Prices of year 0 are expressed in the currency of year 0. Tables 2a- 2c describe the information supplied in national accounts, as a standard. Table 2d is our proposal. Its systematic place is between nominal values and volumes, actually, because account is being taken of monetary variation only, while in the volume table monetary and product price changes have been eliminated, both. Comparing tables 2d and 2c we find that GDP at real values and in volume is identical, be definition, because the set of goods and services comprising GDP has been selected as the numeraire for determining purchasing power parity of the ruling currency. It represents, so to speak, the point of gravity of the economic system, and its movement in monetary space stands for the whole.

For an analysis of other aggregates it is convienent to compile tables of changes in volume and in real price (tables 3). The change in volume has been assumed abritrarily, in order to show its effect on value added. GDP grows by an amount of 20 in product 1, and shrinks by the amount of 10 in product 2. On the production side the assymmetry is amplified. An amount of 20 is added to the output of establishment 1, and 15 is added to value added, while output in product 1 shrinks by 10, and only 5 is lost in in value added of establishment 2. This is the result of double deflation of value added. The resulting volume demonstrates what part of the observed value added change can be attributed to production, either to more employment or to higher productivity. Thus the increase in output of 20 in product 1 can be traced back to a higher input of product 1, a lower input of product 2, and significantly more value added. Both establishments have substited consumption of outside products by own products. Table 3a arrives at a GDP growth rate of 4.0 percent (last line).

Table 3 Changes between year 0 and year 1 (Currency units of year 0)

a) Volume movement

	Prod. 1	Prod. 2	Cons.	Cap. fo.	Export	Import	GDP	Output
Product 1	10	-10	10	-10	10	-10	20	20
Product 2	-5	5	-5	5	-5	5	-10	-10
Value added	15	-5						
Sum			5	-5	5	-5	10	10
perc. of tot.			4,2	-5,0	6,3	-10,0	4,0	1,8

b) Relative price movement

	Prod. 1	Prod. 2	Cons.	Cap. fo.	Export	Import	GDP	Output
Product 1	-2,6	-1,9	-2,2	-1,4	-1,7	-0,7	-4,5	-9,1
Product 2	3,6	3,6	2,3	2,3	1,0	1,0	4,5	11,7
Value added	-10,0	10,0						
Sum			0,12	0,84	-0,70	0,26	0,00	2,59
perc. of tot.			0,08	0,77	-0,74	0,50	0,00	0,47

Value added may grow not only because of a change in conditions of production, but also of market conditions. If output prices outgrow input prices, value added increases without any improvement in production. The relative price movement can be derived by subtracting table 2c volumes from table 2d real values. On the output side establishment 1 experienced a decrease in output value by 9,1, because its price moved upwards more slowly than average. This caused a favorable price movement for input 1, of course, by 2.6, but unfavorable for

product 2 by 3,6. The resulting terms of trade change for value added amounts to -10, which means that the growth in product 1 by 15 has been largely passed on to other sectors of the economy through relatively falling output prices and a deterioration of the terms of trade in favor of establishment 2.

Table 3b shows how price movements have a distributive effect. They decide where an increase in product will be incident when thrown into the bargaining process of the market. Naturally, for the economy as a whole the real price movements add up to nought. Price movements do not create value added, but determine where it is acquired, and in this case there is a reallocation from establishment 1 to 2. Table 3b also shows price indices for subaggregates of GDP. For example, the price index of consumption increases by 0,08 percent compared to GDP, that of export decreases by .74 percent. The terms of trade vis-à-vis the rest of the world have deteriorated, because real value lost through falling exports prices amounts to –.7 and through rising import prices .26, totaling -.96 currency units or 1.2 percent of real exports.

It is clear from table 2 that double deflation rests on the assumption that the volume of GDP (expenditure side) and of value added (production side) are equal. Otherwise the calculation of a volume of value added as the sesidual would not make sense. Table 4 shows that the equality extends to income, double deflated value added equals uniformly deflated income, namely 250. Divide the nominal income of 293 by the price index and 112.3 and you arrive at 260 as compiled in table 2 d. Actually, due to the fact that table 2 contains all information about price and purchasing power movements it is not necessary to provide for a table in real values besides nominal values of income accounts. A simple device is to add the index of the general price level at the top of each table in nominal values giving the user an easy access to full comparability over time if desired. Summing up our argument we write

(8) value added volume = product volume = real income

at market prices, where the equality between real value added and its volume is assured by making GDP the numeraire of the general price level. The reason for the equation 8 is equation 1. If the equality between the three approaches to national accounts holds in nominal terms it must hold in real terms, because in accounting for the purchasing power parity of currency, no value can be either created or lost by definition of the numeraire. On the contrary, introducing the concepts of real value and real prices is a usful means to cope with the problem of variable measure of value, inevitable in a monetary economy.

Table 4 Income accounts

Income generation
(billion Euros)

Year	0	1	Year	0	1
GDP deflator	100	112.7	GDP deflator	100	112.7
Compensation of employees paid	120	130	Value added	250	293
Taxes on production and imports	50	60			
Operating surplus	80	103			

Income distribution

Income distribution

Property income paid	90	100	Operating surplus	80	103
			Taxes on production and imports	50	60
			Compensation of employees received	116	126
			Property income received	104	114
National income	260	303			

It is possible, of course, to go further, and separate the income flows of value added into two components similar to price and volume, wage rate and working hour volumes, for example, but this does not concern us here. Having shown how equality between the three sides is established we finish by doing the necessary algebra.

6. The algebra of deflating national accounts

Equations 5 - 9 describe the triple decomposition of nominal value change of a national accounts item into three components of volume, and real price of the product, and purchasing power parity of the currency in the ideal case of continuous movement in time. As infinitesimal time intervals are not operable in statistics, an approximation must be employed for working with finite time intervals and changes of variables. Before doing so it is convenient to deal the problem of aggregation, in differential terms, too, because it incorporates the definition of the purchasing power parity index, the general price level.

Let product i be an elementary item of a classification of products i = 1,..., n. The aggregate variables of value decomposition are then defined by summation,

(10) $$dw = \Sigma\, dw_i = (\Sigma\lambda\, \pi_i\, dq_i + \Sigma\lambda\, q_i\, d\pi_i + \Sigma\pi_i\, q_i\, d\lambda)\, w_i^{\,0}\,.$$

Because of equations 5 to 9 this can also be written as

(11) $$\frac{dw}{w} = \sum \frac{w_i}{w}\frac{dq_i}{q_i} + \sum \frac{w_i}{w}\frac{d\pi_i}{\pi_i} + \frac{d\lambda}{\lambda}$$

which leads to the corresponding decomposition at the aggregate level

(12) $$\frac{dq}{q} = \sum \frac{w_i}{w}\frac{dq_i}{q_i}\,,$$

(13) $$\frac{d\pi}{\pi} = \sum \frac{w_i}{w}\frac{d\pi_i}{\pi_i}\,.$$

Equations 12 and 13 may be recognized as the Divisia formula for a continuous index of aggregate price and volume movements. Expressed in values of the base year, equations 11, 12, an 13 yield

(14) $$dw = (dq + d\pi + d\lambda)\, w^0\,,$$

where variables without subindex stand for aggregates, and are defined within equation 10.

A special case occurs if the aggregate in question is the numeraire selected for determining the purchasing power parity changes of the national currency. The nominal price change of this aggregate is fully attributed to the monetary side, by definition. Hence we have

(15) $$\frac{d\lambda}{\lambda} = \sum \frac{w_i}{w} \frac{dp_i}{p_i} \quad .$$

Because of equation 5 this implies

(16) $$\frac{d\lambda}{\lambda} = \sum \frac{w_i}{w} \frac{d\pi_i}{\pi_i} - \sum \frac{w_i}{w} \frac{d\lambda}{\lambda}$$

and hence

(17) $$\frac{d\pi}{\pi} = 0$$

the aggregate real price change of the numeraire is zero, be definition.

Now we approximate the differentials of equation 10 by finite differences,

(18) $$\Delta w = \Sigma \Delta w_i = (\Sigma \lambda \pi_i \Delta q_i + \Sigma \lambda q_i \Delta \pi_i + \Sigma \pi_i q_i \Delta \lambda) w_i^{0}$$

where Δ denotes the difference between a base period 0 and the period 1 following it. The is no unique way of approximating these differentials. The history of the index number problem witnesses many a controvery on the issue. A practical decomposition is given by the following equation,

(19) $$w_i^1 - w_i^0 = \left[\lambda^0 \pi_i^0 \left(q_i^1 - q_i^0\right) + \lambda^0 q_i^1 \left(\pi_i^1 - \pi_i^0\right) + \pi_i^1 q_i^1 \left(\lambda^1 - \lambda^0\right)\right] w_i^0 \ ,$$

where the base year values of the ewlementary indexes are

(20) $$q_i^0 = \pi_i^0 = \lambda_i^0 = 1,$$

by definition. Formula 19 implies the Laspeyres index for aggregate volume change, and the Paasche index for aggregate real price and price level changes. These indexes have been used in compiling tables 3. Hillinger (1999) shows that an approximation of second order may be obtained by relying on the arithmetic average of Paasche and Laspeyres indexes.

If more than two periods are to be compared modern chaining is applied, which means that the pricel level is chained together from Paasche or Laspeyres links, or their arithmetic average, transforming nominal values of each year into real values expressed in purchasing power parity fo a base year, and the ensuing decomposition of these real values into volume and real price is even additive, if performed according to the principles, elaborated above. To prove this is not the topic here.

However, a final remark on the SNA is in order. If equation 8 is true, equation 4 is not. Trading gains must not be subtracted from GDP at constant prices in order to yield domestic income. For if GDP is used as the numeraire for purchasing power parity we have

(21) $$\frac{GDP}{P} = \frac{C}{P_C} + \frac{I}{P_I} + \frac{X}{P_x} - \frac{M}{P_m} = \frac{Y}{P} \ .$$

In words, the volume of GDP being defined as the sum of the volume of its components yields an implicit price index P for measuring the purchasing power parity inherent in domestic income Y. Equation 21 may be transformed into

(22) $$\left(\frac{C}{P} - \frac{C}{P_C}\right) + \left(\frac{I}{P} - \frac{I}{P_I}\right) + \left(\frac{X}{P} - \frac{X}{P_X}\right) - \left(\frac{M}{P} - \frac{M}{P_M}\right) = 0 \,,$$

demonstrating that the trading gain or loss incurred in international business according to equation 3 is compensated by an opposite movement of domestic real prices, due to the relativity of purchasing power parity measurement. The argument would also hold, although less evidently, if some other "broad measure" of purchasing power parity is applied. But, of course, the real value of all GDP rises and falls with the effective real exchange rate of the national currency.

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